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HWBP voor sterke dijken

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memo Spreadsheets voor parameterbepaling

Inleiding

Voor het opstellen van proevenverzamelingen, die nodig zijn om parameters te bepalen voor geotechnische stabiliteitsanalyses van dijken, worden verschillende methodes toegepast. Deze memo is bedoeld om een overzicht te geven van de verschillende op dit moment beschikbare methoden en het ontsluiten van bijbehorende spreadsheets, zodat deze in projecten eenduidig kunnen worden toegepast of kunnen worden gebruikt om in projecten opgestelde script te verifiëren.

In deze memo worden de op dit moment bij DIV bekende methoden behandeld. Wanneer nieuwe/andere methoden beschikbaar komen/zijn kunnen deze aan deze memo worden toegevoegd.

Allereerst wordt kort ingegaan op het verschil tussen een lokale en een regionale proevenverzameling. Vervolgens worden de verschillende beschikbare methoden toegelicht. Aan het eind van deze memo wordt nog een nawoord en overzicht van de referenties gegeven.

Lokale of regionale proevenverzameling

Bij waterkeringen wordt ruimtelijke spreiding over het algemeen in rekening gebracht met een spreidingsreductiefactor. Hierbij is het van belang of er sprake is van een lokale of regionale proevenverzameling.

Voor macrostabiliteit wordt bij een *lokale* proevenverzameling een spreidingsreductiefactor van $\Gamma = 0$ toegepast. De verhouding tussen de lokale en regionale variantie is dan $\alpha = 1{,}0$ ($\alpha \approx 1 - \Gamma^2$), omdat er geen rekening wordt gehouden met regionale variantie. Hierbij wordt ervan uitgegaan dat:

- De *gemiddelde* sterkte langs het glijvlak relevant is voor het optreden van het faalmechanisme.

- De uitgevoerde proeven binnen de omvang van een glijvlak zijn uitgevoerd. Over het algemeen is dit binnen een gebied van ca. 25-100m.
- De proeven verspreid over dit gebied zijn uitgevoerd.

Voor macrostabiliteit wordt bij een *regionale* proevenverzameling wordt over het algemeen een spreidingsreductiefactor van Γ = 0,5 toegepast. Dit komt ongeveer overeen met een verhouding tussen de lokale en regionale variantie van α = 0,75 ($\alpha \approx 1 - \Gamma^2$). Deze waarde kan echter ook projectspecifiek worden afgeleid. Bij een waarde van Γ = 0,5 wordt ervan uitgegaan dat:

- De *gemiddelde* sterkte langs het glijvlak relevant is voor het ontstaan van het faalmechanisme.
- De ruimtelijke spreiding verticaal over een grondlaag wordt uitgemiddeld en de ruimtelijke spreiding langs de dijk niet wordt uitgemiddeld.
- De proeven verspreid over het gebied zijn uitgevoerd.

Voor meer informatie en achtergronden wordt verwezen naar o.a. bijlage 2, [TRWG 2001] en [SHM 2021].

Spreadsheets voor parameterbepaling

In dit hoofdstuk worden de verschillende beschikbare statische methoden en bijbehorende spreadsheets behandeld. Een overzicht is weergegeven in Tabel 1.

Bij het Hierbij wordt allereerst ingegaan op het bepalen van de parameters van een kansverdeling door deze te fitten op de data uit een proevenverzameling. Hierbij is het uitgangspunt dat de parameter onafhankelijk is van andere (grond)parameters. Vervolgens worden een aantal methoden beschreven, waarbij de statistische analyse op basis van lineaire regressie plaatsvindt. Hierbij is het uitgangspunt dat er een lineair verband bestaat tussen twee parameters.

Tabel 1: Overzicht spreadsheets

Bijlage	Parameter(s)	Statistische methode	Type proeven	Aannames
4	S, m, POP, φ_{cs}, …	Fitten kansverdeling	TX of DSS of OED/CRS	Parameters zijn onafhankelijk van elkaar.
5	c' en φ'	Lineaire regressie	TX	Schuifsterkte >0 bij effectieve spanning = 0. Lineair verband tussen effectieve spanning en schuifsterkte
6	c' en φ'	Lineaire regressie	DSS	Schuifsterkte >0 bij effectieve spanning = 0. Lineair verband tussen effectieve spanning en schuifsterkte
7	S en m	Lineaire regressie	TX en/of DSS	OCR bekend (bijv. uit OED proef). Lineair verband tussen ln(OCR) en $\ln(s_u/\sigma'_{v;0})$.
8	S en POP	Lineaire regressie	TX en/of DSS	m bekend (bijv. o.b.v. samendrukkingsproef of literatuur). Lineair verband tussen effectieve spanning en schuifsterkte.
9	Su-tabel en m	Lineaire regressie	TX en/of DSS	Lineair verband tussen $\ln(\sigma'_{v;0})$ en $\ln(s_u)$.

TX = triaxiaalproef, DSS = DSS-proef, OED = oedometerproef, CRS = Constant Rate of Strain proef

c' = effectieve cohesie, φ' = effectieve hoek van inwendige wrijving
S = normaal geconsolideerde ongedraineerde schuifsterkte ratio, m = sterktetoename-exponent, POP = Pre Overburden Pressure, OCR = Over Consolidation Ratio
s_u = ongedraineerde schuifsterkte, $\sigma'_{v;0}$ = effectieve verticale consolidatiespanning

Fitten kansverdeling: bepaling parameters (log)normale verdeling uit dataset (bijv. S, m, POP, φ_{cs})

Als er geen verband is tussen bepaalde parameters, kunnen de parameters van een (log)normale verdeling worden bepaald door een kansverdeling te fitten aan de dataset. Dit kan bijvoorbeeld het geval zijn voor S, m, POP, φ_{cs}, aangezien deze in principe niet afhankelijk zijn van andere parameters, zoals het spanningsniveau.

De spreadsheet om de gefitte parameters van de (log)normale verdeling en de karakteristieke waarde van de variabele te bepalen is bijgevoegd als bijlage 4 bij deze memo. De formules zijn gebaseerd op [CUR 190], bijlage 2 en bijlage 3. Een overzicht van de rekenmethoden om de parameters voor de kansverdelingen en de karakteristieke waarden te bepalen is weergegeven in Tabel 2

Tabel 2: Overzicht rekenmethoden

#	Type verdeling	Fysische minimum	Karakteristieke ondergrenswaarde van …
1	Normaal	n.v.t.	de parameter
2	Normaal	n.v.t.	het (laag)gemiddelde van de parameter
3	Lognormaal	n.v.t.	de parameter
4	Lognormaal	n.v.t.	de mediaan van de parameter
5	Lognormaal	n.v.t.	het (laag)gemiddelde van de parameter
6	Lognormaal	Ja	de parameter
7	Lognormaal	Ja	de mediaan van de parameter
8	Lognormaal	Ja	het (laag)gemiddelde van de parameter

Lineaire regressie: bepaling c' en φ' uit eentraps triaxiaalproeven

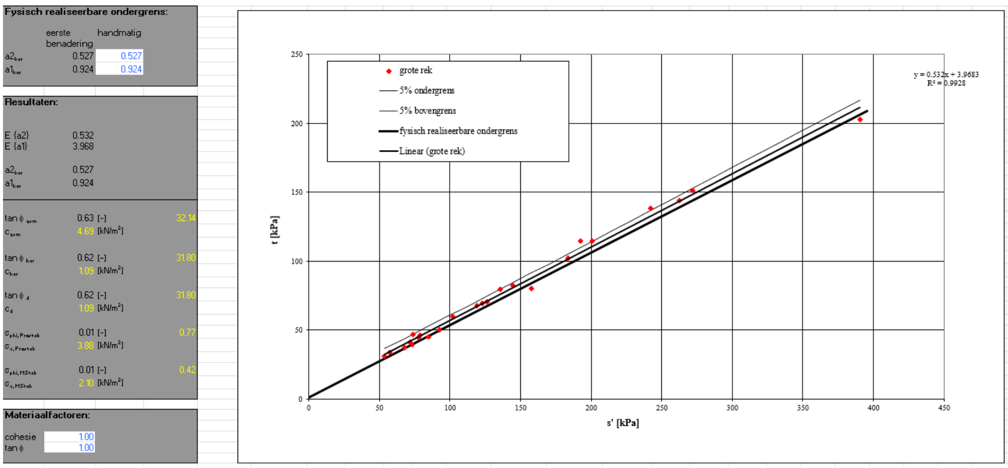
Voor het opstellen van proevenverzamelingen van de schuifsterkte parameters cohesie c' en hoek van inwendige wrijving φ' voor stabiliteitsanalyses met het Mohr-Coulomb model is in het verleden door Deltares een spreadsheet opgesteld. Het spreadsheet berekent verwachtingswaarden en karakteristieke waarden op basis van een serie proefresultaten uit triaxiaalproeven doormiddel van lineaire regressie. Hierbij is aangenomen dat c' en φ' constanten zijn met een bandbreedte, welke onafhankelijk zijn van het spanningsniveau. Hierbij kan onderscheid worden gemaakt in een lokale of regionale proevenverzameling. Voor meer details wordt verwezen naar bijlage 1.

De lineaire regressie is gebaseerd op de volgende relatie:
Y = a2.X+a1

Hierin is:

Y = t (schuifspanning of halve deviatorspanning $(\sigma'_1-\sigma'_3)/2$)	[kPa]
X = s' (gemiddelde effectieve spanning $(\sigma'_1+\sigma'_3)/2$)	[kPa]
a2 = helling, sin(φ')	[-]
a1 = intercept, c'.cos(φ')	[kPa]

De spreadsheet is als bijlage 5 bij deze memo gevoegd. Een voorbeeld van het resultaten van deze spreadsheet 'Bijlage5_C-Phi_TX_PV.xls' is weergegeven in figuur 1.



figuur 1: Voorbeeld resultaat c'-φ' bepaling uit triaxiaalproeven

In de spreadsheet is het ook mogelijk om de lineaire regressie 'handmatig' uit te voeren. Dit kan bijvoorbeeld worden gedaan om fysisch onrealistische waarden te voorkomen.

Lineaire regressie: bepaling c' en φ' uit DSS proeven

Vergelijkbaar met de beschikbare spreadsheet voor het bepalen van c' en φ' uit triaxiaalproeven is een spreadsheet beschikbaar voor het afleiden van deze parameters uit DSS proeven. Ook hier is de aanname dat c' en φ' constanten zijn met een bandbreedte, welke onafhankelijk zijn van het spanningsniveau.

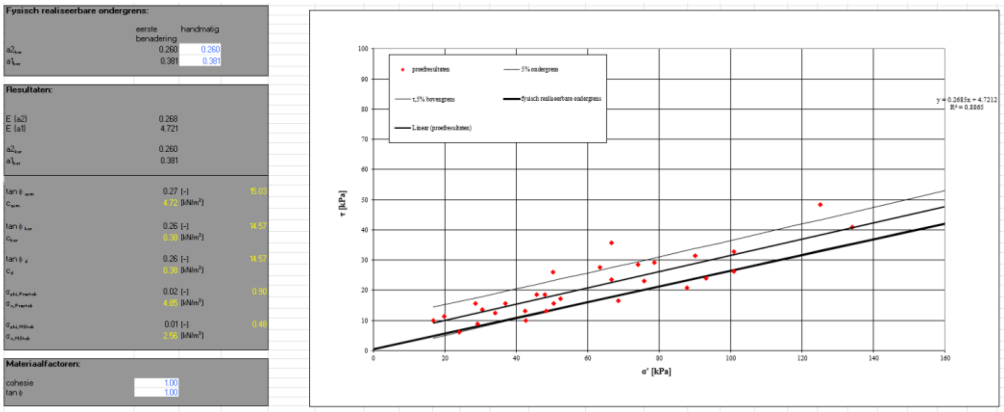
De lineaire regressie is gebaseerd op de volgende relatie:
Y = a2.X+a1

Hierin is:

Y = τ, ongedraineerde schuifsterkte s_u	[kPa]
X = verticale effectieve spanning σ'_v	[kPa]
a2 = helling, tan(φ')	[-]
a1 = intercept, c'	[kPa]

Hierbij is het belangrijk om op te merken dat de bepaling van de hoek van inwendige wrijving uit een DSS-proef afhankelijk is van de aanname of associatief of niet-associatief grondgedrag optreedt. Wanneer uit wordt gegaan van associatief grondgedrag (ψ=φ'), geldt $\varphi' = \tan^{-1}(\tau/\sigma'_v)$. Dit is ook de aanname die aan deze sheet ten grondslag ligt. Wanneer echter wordt uitgegaan van niet-associatief grondgedrag (ψ=0) -zoals gebruikelijk is in het WBI en BOI-, geldt $\varphi' = \sin^{-1}(\tau/\sigma'_v)$. Als uit wordt gegaan van niet-associatief grondgedrag kan dezelfde sheet als voor de bepalen van c' en φ' uit triaxiaalproeven worden gebruikt, waarbij s' wordt vervangen door σ'_v en t wordt vervangen door τ.

De spreadsheet (voor associatief grondgedrag) is als bijlage 6 bij deze memo gevoegd. Een voorbeeld van het resultaten van deze spreadsheet 'Bijlage6_C-Phi_DSS_PV' is weergegeven in figuur 2.



figuur 2: Voorbeeld resultaat c'-φ' bepaling uit DSS proeven

In de spreadsheet is het ook mogelijk om de lineaire regressie 'handmatig' uit te voeren. Dit kan bijvoorbeeld worden gedaan om fysisch onrealistische waarden te voorkomen.

Lineaire regressie: bepaling S en m uit triaxiaal- of DSS proeven

De beschikbare spreadsheet voor het bepalen van c' en φ' uit triaxiaal- of DSS proeven is ook geschikt om S en m ten behoeve van het SHANSEP-model af te leiden uit triaxiaal- en DSS-proeven. Hierbij worden proeven waarbij de overconsolidatieratio (OCR) is opgelegd gebruikt, zodat de afhankelijkheid van de sterkte van de OCR doormiddel van de parameter m kan worden bepaald.

De lineaire regressie is gebaseerd op de volgende relatie:

$$Y = a2.X+a1$$

Hierin is:

Y = natuurlijke logaritme ongedraineerde schuifsterkte s_u gedeeld door verticale effectieve consolidatiespanning $\sigma'_v = \ln(s_u/\sigma'_v)$ [kPa]

X = natuurlijke logaritme van de overconsolidatieratio OCR = ln(OCR) [-]

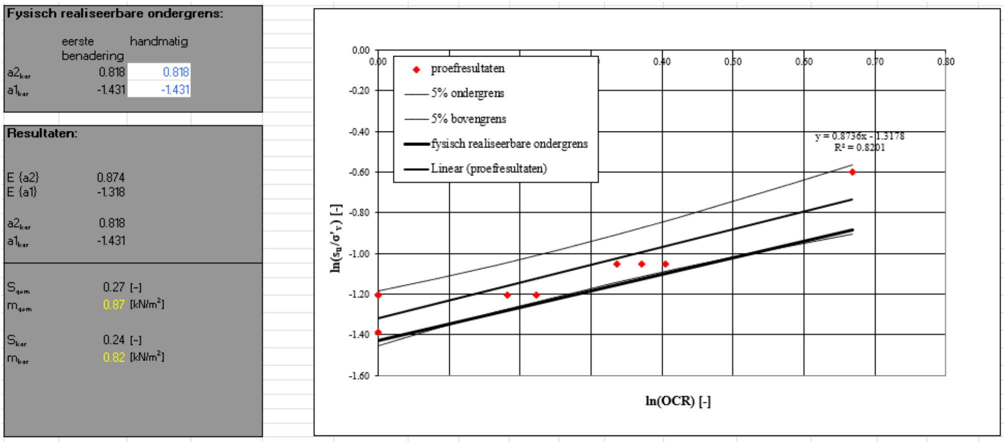
a2 = helling, m [-]

a1 = intercept, natuurlijke logaritme normaal geconsolideerde schuifsterkteratio = ln(S) [-]

Hierbij is de SHANSEP formulering als volgt omgeschreven:

$s_u = S.\sigma'_v.OCR^m$	[kPa]
$s_u/\sigma'_v = S.OCR^m$	[kPa]
$\ln(s_u/\sigma'_v) = \ln(S) + m.\ln(OCR)$	[-]

De spreadsheet is als bijlage 7 bij deze memo gevoegd. Een voorbeeld van het resultaten van deze spreadsheet 'Bijlage7_SHANSEP-S-m_PV' is weergegeven in figuur 3.



figuur 3: Voorbeeld resultaat S-m bepaling

In de spreadsheet is het ook mogelijk om de lineaire regressie 'handmatig' uit te voeren. Dit kan bijvoorbeeld worden gedaan om fysisch onrealistische waarden te voorkomen.

Lineaire regressie: bepaling S en POP uit triaxiaal- of DSS proeven

De beschikbare spreadsheet voor het bepalen van c' en φ' uit triaxiaal- of DSS proeven is ook geschikt gemaakt om S en POP ten behoeve van het SHANSEP-model af te leiden uit triaxiaal- en DSS-proeven. Hiervoor worden proeven die bij in-situ spanning zijn uitgevoerd gebruikt, zodat de POP onder 'dagelijkse omstandigheden' kan worden bepaald. Uitgangspunt voor de spreadsheet is dat de sterktetoename-exponent m als 'bekend' wordt verondersteld. Deze kan worden geschat op basis van kentallen of op basis van samendrukkingsproeven. Hierbij is aangenomen dat S en POP constanten zijn met een bandbreedte, welke onafhankelijk zijn van het spanningsniveau.

De lineaire regressie is gebaseerd op de volgende relatie:

$Y = a2.X+a1$

Hierin is:

Y = ongedraineerde schuifsterkte s_u	[kPa]
X = verticale in-situ effectieve spanning = verticale effectieve consolidatiespanning σ'_v	[kPa]
a2 = helling, S	[-]
a1 = intercept, S.m.POP	[kPa]

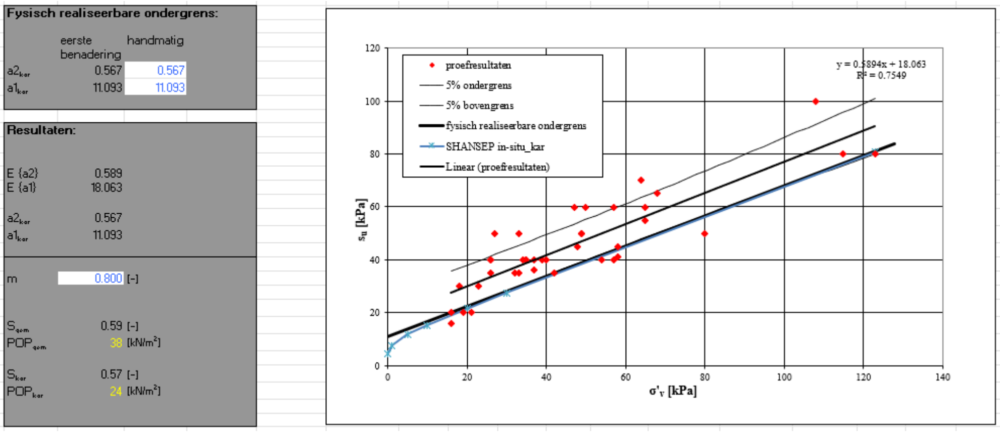
Hierbij is de SHANSEP formulering als volgt omgeschreven:

$s_u = S.\sigma'_v.((POP+\sigma'_v)/\sigma'_v)^m$ [kPa]

$s_u \approx S.\sigma'_v+S.m.POP$ voor $POP < \sigma'_v$ [kPa]

Deze vergelijking benaderd de SHANSEP formulering in de limiet voor POP is nul, maar het verschil is beperkt tot enkele procenten wanneer de POP kleiner is dan de verticale in-situ effectieve spanning. Bij een grotere POP (of kleinere spanning) is het verschil met de lineaire benadering groter. In de sheet is het resulterende verband tussen in-situ spanning en in-situ sterkte ook weergegeven, zodat te controleren is of de statistische analyse beïnvloed wordt door deze afwijking. Dit is het geval als de proefresultaten in het gebied liggen waar de lijn die de in-situ sterkte als functie van de in-situ spanning op basis van het SHANSEP-model (blauwe lijn in figuur 4) afwijkt van de linearisatie (dikke zwarte lijn in figuur 4).

De spreadsheet is als bijlage 8 bij deze memo gevoegd. Een voorbeeld van het resultaten van deze spreadsheet 'Bijlage8_SHANSEP-S-POP_PV' is weergegeven in figuur 4.



figuur 4: Voorbeeld resultaat S-POP bepaling

Deze methode geeft vaak ook goede resultaten voor zandige kleien en/of transitional soils, waardoor het gebruik van een su-tabel niet nodig is. Ook is het met deze aanpak niet nodig om de grensspanning of POP uit samendrukkingsproeven af te leiden, wat bijvoorbeeld voor transitional soils vaak niet eenduidig is. *Noot: het is wel mogelijk om een s_u-tabel toe te passen, waarbij de 'blauwe' lijn uit figuur 4 als s_u-tabel wordt gehanteerd.* Het voordeel van het toepassen van de SHANSEP formulering is dat deze in meer software pakketten ondersteund wordt. Het is ook mogelijk om de normaal geconsolideerde S ratio uit sterkteproeven af te leiden en deze te vergelijken met de waarde uit de regressieanalyse. Deze kan dan eventueel handmatig worden aangepast als daar aanleiding toe is.

Zoals eerder aangegeven kan de waarde van m worden bepaald uit samendrukkingsproeven. Als deze niet beschikbaar zijn, is een waarde van 0,7-0,8 een realistische inschatting voor zandige klei, 0,85-0,95 voor veen en humeuze klei en 0,8-0,9 voor overige kleien. Het is belangrijk om te realiseren dat voor ontlastsituaties (bijv. stijging freatische lijn) een lage waarde tot een lagere sterkte leidt en voor belastsituaties (bijv. ophogingen) een hoge waarde tot een lagere sterkte leidt.

Aandachtspunt is dat -als een relatief hoge POP wordt gevonden- bij lage spanningen (ongeveer tot een spanningsniveau gelijk aan de POP, ofwel OCR=2) relatief hoge sterktes kunnen worden gevonden als een realistische m van ca. 0,7-0,9 wordt toegepast. Gecontroleerd moet worden of deze sterktes realistisch zijn, met name bij ondiepe glijvlakken is dit een belangrijk aandachtspunt. In deze zone speelt vaak ook de invloed van structuurvorming een rol bij dijksmateriaal. Ook als er sprake is van een opdrijfzone is sprake van lage spanningen en moet de sterkte bij deze lage spanningen met zorg worden gekozen. In die gevallen kan er voor gekozen worden om de ongedraineerde schuifsterkte in het geotechnisch rekenmodel te begrenzen.

In de sheet is het ook mogelijk om de lineaire regressie 'handmatig' uit te voeren. Dit kan bijvoorbeeld worden gedaan om fysisch onrealistische waarden te voorkomen.

Lineaire regressie: bepaling s_u-tabel en m uit triaxiaal- of DSS proeven

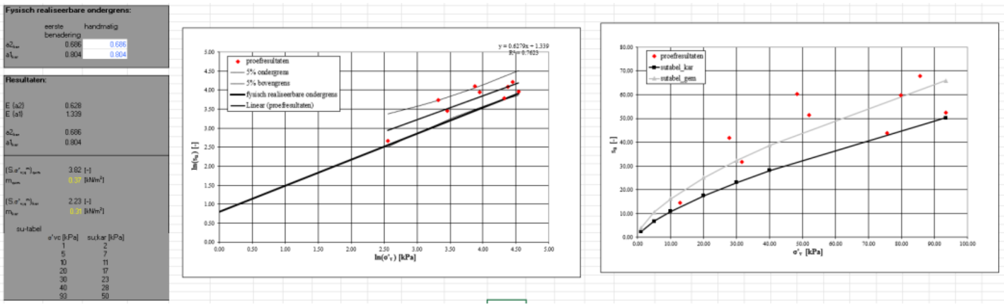
De beschikbare spreadsheet voor het bepalen van c' en φ' uit triaxiaal- of DSS proeven is ook geschikt gemaakt om een s_u-tabel en m af te leiden uit triaxiaal- en DSS proeven. Hiervoor worden proeven die bij in-situ spanning zijn uitgevoerd gebruikt, zodat de spanningsafhankelijkheid van de van de spanning doormiddel van m kan worden bepaald.

De lineaire regressie is gebaseerd op de volgende relatie:
$Y = a2.X+a1$

Hierin is:
Y = natuurlijke logaritme ongedraineerde schuifsterkte $s_u = \ln(s_u)$ [kPa]
X = natuurlijke logaritme verticale in-situ effectieve spanning = verticale effectieve consolidatiespanning = $\ln(\sigma'_v)$ [kPa]
a2 = helling, 1-m [-]
a1 = intercept, natuurlijke logaritme normaal geconsolideerde schuifsterkteratio maal grensspanning tot de macht m = $\ln(S.\sigma'_{v;g}{}^m)$ [kPa]

Hierbij is de SHANSEP formulering als volgt omgeschreven:
$s_u = S.\sigma'_v.(\sigma'_{v;g}/\sigma'_v)^m = S.\sigma'_{v;g}{}^m.\sigma'_v{}^{(1-m)}$ [kPa]
$\ln(s_u) = (1-m).\ln(\sigma'_v)+\ln(S.\sigma'_{v;g})^m$ [kPa]

De spreadsheet is als bijlage 9 bij deze memo gevoegd. Een voorbeeld van het resultaten van deze spreadsheet 'Bijlage9_SHANSEP-sutabel-m_PV' is weergegeven in figuur 5.



figuur 5: Voorbeeld resultaat su-tabel-m bepaling

Met deze aanpak is het niet nodig om een grensspanning uit samendrukkingsproeven of ongedraineerde schuifspanningsratio uit normaal geconsolideerde sterkteproeven af te leiden. Overigens is het ook mogelijk door ófwel de grensspanning ófwel de ongedraineerde schuifspanningsratio bekend te veronderstellen (of verstandig te kiezen) om in plaats van de s_u-tabel de SHANSEP formulering toe te passen. Hierbij geldt de grensspanning als modelparameter en niet de POP of OCR. Ook hier geldt het aandachtspunt dat uit het model bij lage spanningen (ongeveer tot een spanningsniveau gelijk aan de POP, ofwel OCR=2) relatief hoge sterktes kunnen worden gevonden. Gecontroleerd moet worden of deze sterktes realistisch zijn, met name bij ondiepe glijvlakken is dit een belangrijk aandachtspunt. In deze zone speelt vaak ook de invloed van structuurvorming een rol bij dijksmateriaal. Ook als er sprake is van een opdrijfzone is sprake van lage spanningen en moet de sterkte bij deze lage spanningen met zorg worden gekozen.

In de praktijk blijken vaak onrealistisch lage m waarden gevonden te worden met deze methode. Dit is vaak het gevolg van de impliciete aanname dat de grensspanning (of eigenlijk $S.\sigma'_g{}^m$) een constante is (met bandbreedte). Het is dan vaak realistischer om een hogere m te hanteren in combinatie met een constante POP (met bandbreedte). Hierbij kan bijvoorbeeld de regressieanalyse uit de voorgaande paragraaf worden toegepast.

In de sheet is het ook mogelijk om de lineaire regressie 'handmatig' uit te voeren. Dit kan bijvoorbeeld worden gedaan om fysisch onrealistische waarden te voorkomen.

Nawoord

Het is niet zo dat één bepaalde statistische methode en bijbehorende spreadsheet altijd het 'beste' resultaat oplevert. Uiteindelijk hangt dit ook samen met het gekozen materiaalmodel en het gedrag of de belastinghistorie van het specifieke materiaal. Dit kan er bijvoorbeeld toe leiden dat in een bepaalde situatie het hanteren van een constante (maar wel variabele) POP realistischer is en in een andere situatie juist een constante (maar wel variabele) OCR of grensspanning. Een hulpmiddel hierbij kan door de statistische analyse met verschillende methoden uit te voeren en te bepalen welk model met bijbehorende parameters het beste aansluit bij de data. Het is uiteindelijk aan de geotechnisch adviseur om het best passende model met bijbehorende parameterset te bepalen, welke het best aansluiten op de fysica en waarmee recht wordt gedaan aan onzekerheden.

Bij vragen over het toepassen van deze memo (of bijbehorende spreadsheets), het delen van ervaringen en/of bij tips voor verbetering of uitbreiding, kan contact worden opgenomen met DIV via **div@wsrl.nl**.

Referenties

[CUR 190] Kansen in de Civiele Techniek – Deel 1: Probabilistische Ontwerpen in de Theorie, Civieltechnisch Centrum Uitvoering Research en Regelgeving, maart 1997

[SHM 2021] Schematiseringshandleiding Macrostabiliteit, Ministerie van Infrastructuur en Waterstaat, 28 mei 2021

[TRWG 2001] Technisch Rapport Waterkerende Grondconstructies, Technische Adviescommissie Waterkeringen, 2001

Bijlage 1: Memo ‘Bepaling karakteristieke waarden schuifsterkte parameters’ (Deltares)

Memo

Aan
Sander Kapinga (Waterschap Rivierenland)

Datum	**Kenmerk**	**Aantal pagina's**
24 februari 2016	1220132-003-GEO-0002	14
Van	**Doorkiesnummer**	**E-mail**
Ed Calle, Alexander van Duinen	+31(0)88335 7201	alexander.vanduinen@deltares.nl

Onderwerp
Bepaling karakteristieke waarden schuifsterkte parameters

1. Inleiding

Waterschap Rivierenland is in 2013 gestart met de verkenningsfase voor de HWBP koploperprojecten. Deze verkenningsfase moet resulteren in een voorkeursalternatief voor de (eventueel noodzakelijke) dijkversterkingen. Waterschap Rivierland (WSRL) heeft aan Deltares gevraagd om bij de uitvoering van het fase 2 grondonderzoek de kwaliteitscontrole uit te voeren op het specialistisch grondonderzoek zowel in het veld als in het laboratorium. Tevens verzorgt Deltares voor WSRL de kennisoverdracht, begeleiding en controle bij het opstellen van de proevenverzamelingen en correlaties van de ondergrondparameters.

Voor het opstellen van proevenverzamelingen van de schuifsterkte parameters cohesie c' en hoek van inwendige wrijving ϕ' voor gedraineerde stabiliteitsanalyses is sinds een aantal jaren een spreadsheet in gebruik bij Deltares. Het spreadsheet wordt ook gebruikt door andere partijen, bijvoorbeeld in de database proevenverzameling die door Hoogheemraadschap Hollands Noorderkwartier en Waterschap Rivierenland is ontwikkeld. Het spreadsheet is deels gebaseerd op Van Esch (1997) en aangevuld met nieuwere inzichten. Een complete beschrijving van het spreadsheet ontbrak tot nu toe. Waterschap Rivierenland heeft verzocht een beschrijving van de achtergronden van dit spreadsheet op te stellen. Deze beschrijving is opgenomen in de voorliggende notitie. Daarnaast wordt in deze notitie ingegaan op een uitbreiding van het oorspronkelijke spreadsheet voor toepassing voor ongedraineerde schuifsterkte parameters. Het spreadsheet is verkrijgbaar via de Helpdesk Water.

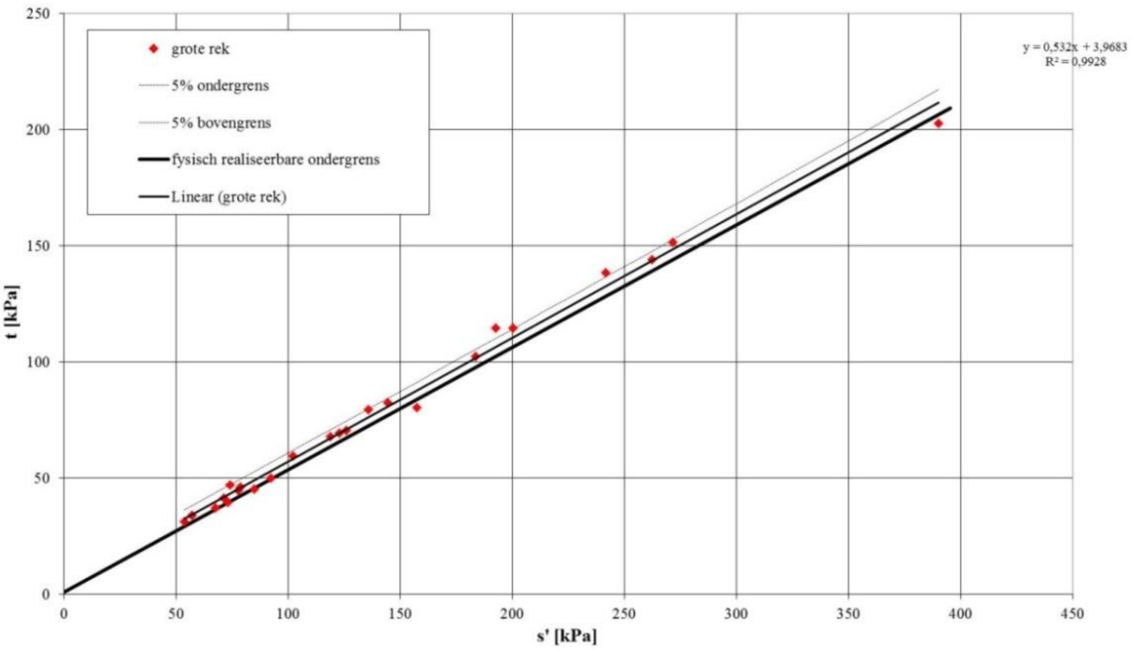
2. Toepassingen in spreadsheets

2.1 *c' en tan(ϕ') voor gedraineerde analyses*

Het spreadsheet berekent verwachtingswaarden, karakteristieke waarden en rekenwaarden van de cohesie c' (kPa) en hoek van inwendige wrijving ϕ' (º) op basis van een serie proefresultaten uit triaxiaalproeven of direct simple shear proeven (zie Figuur 2.1). Hiervoor wordt uitgegaan van de relaties die worden beschreven in hoofdstuk 3. De invoerwaarden uit triaxiaalproeven zijn de gemiddelde spanning s' ($(\sigma'_1+\sigma'_3)/2$ in kPa) en de schuifspanning t ($(\sigma'_1-\sigma'_3)/2$ in kPa). De invoerwaarden uit direct simple shear proeven zijn de verticale spanning σ_v (kPa) en de schuifspanning τ (kPa).

Bij het uitwerken van de proevenverzameling kan in het spreadsheet onderscheid worden gemaakt tussen een lokale of regionale proevenverzameling. Voor een lokale proevenverzameling is α gelijk aan 1,0. Voor een regionale proevenverzameling dient α in beginsel bepaald te worden aan de hand van ruimtelijk statistische analyse van de proevenverzameling. In (Calle 2007 en 2008) is hiervoor een ruimtelijke statistische methode beschreven, die overigens nogal specialistisch is. In het verleden is de α voor regionale proevenverzamelingen van gedraineerde schuifsterkteparameters gelijk aan 0,75 gekozen (zie TAW, 1989). Dit was een redelijk geachte, maar wel arbitraire, keuze, die sindsdien vaak is aangehouden. De veel latere statistische analyses in (Calle, 2007 en 2008), die overigens niet een regionale proevenverzameling van schuifsterkteparameters, maar van zettingsparameters betrof, gaf niet veel aanleiding om de vroegere keuze te herzien of nader te onderbouwen.

Het kan voorkomen dat de met de kleinste kwadratenmethode berekende ondergrens met 5% onderschrijdingskans tot negatieve waarden van de karakteristieke waarde van de cohesie c' leidt. In dat geval kan de karakteristieke waarde van de variabele $\hat{a}_1$ handmatig op 0 worden gesteld en kan de karakteristieke waarde van de variabele $\hat{a}_2$ zodanig worden aangepast dat de ‘fysisch realiseerbare ondergrens’ (zie Figuur 2.1) goed overeenkomt met de ondergrens die met de kleinste kwadratenmethode is berekend. De variabelen $\hat{a}_1$ en $\hat{a}_2$ zijn regressieparameters die worden bepaald volgens de kleinste kwadratenmethode (zie hoofdstuk 3).



Figuur 2.1 Fysisch realiseerbare ondergrens en het gebied met 90% betrouwbaarheid.

2.2 Ongedraineerde schuifsterkte

Voor de bepaling van de karakteristieke waarden van de ongedraineerde schuifsterkte parameters S en m wordt als uitgangspunt genomen dat $\log(s_u/\sigma'_{vc})$ normaal verdeeld is, en dus log(S) en m bepaald kunnen worden via lineaire regressie op basis van meetwaarden van $\log(s_u/\sigma'_{vc})$ versus log(OCR). Of dit uitgangspunt juist is voor toepassing bij ongedraineerde schuifsterkteparameters, moet formeel aan de hand van statistische toetsen op beschikbare gegevens over de schuifsterkteparameters worden vastgesteld.

In de kleinste-kwadraten-aanpak is (vrijwel exacte) instelbaarheid van de “x”-variabele uitgangspunt. Wanneer onzekerheid over die variabele, ten opzichte van de onzekerheid van de afhankelijke “y”-variabele, substantieel is, dan kan de kleinste kwadraten-analyse strikt genomen niet worden toegepast. In deze notitie wordt er van uitgegaan dat de ongedraineerde schuifsterkte s_u en de ongedraineerde schuifsterkte ratio s_u/σ'_{vc} worden bepaald op grondmonsters die zijn geconsolideerd bij de in situ verticale effectieve spanning σ'_{vi} (zogenaamde recompression of “Norwegian style” procedure). Toepassing van deze werkwijze is economisch als ook een correlatie wordt opgesteld tussen de ongedraineerde schuifsterkte en de sondeerweerstand. Voordeel van deze werkwijze is ook dat de grondmonsters in de proeven tijdens de consolidatie niet worden belast met een hoge spanning, zodat de structuur die de grond heeft opgebouwd in het veld zoveel mogelijk intact blijft. Nadeel van deze werkwijze is echter dat de schuifsterkte en de grensspanning en OCR noodzakelijkerwijze worden bepaald op verschillende monsters (van mogelijk iets verschillende samenstelling en historie). Er is dus onzekerheid over OCR. De toepasbaarheid van de kleinste kwadratenmethode is in dit geval dus niet vanzelfsprekend.

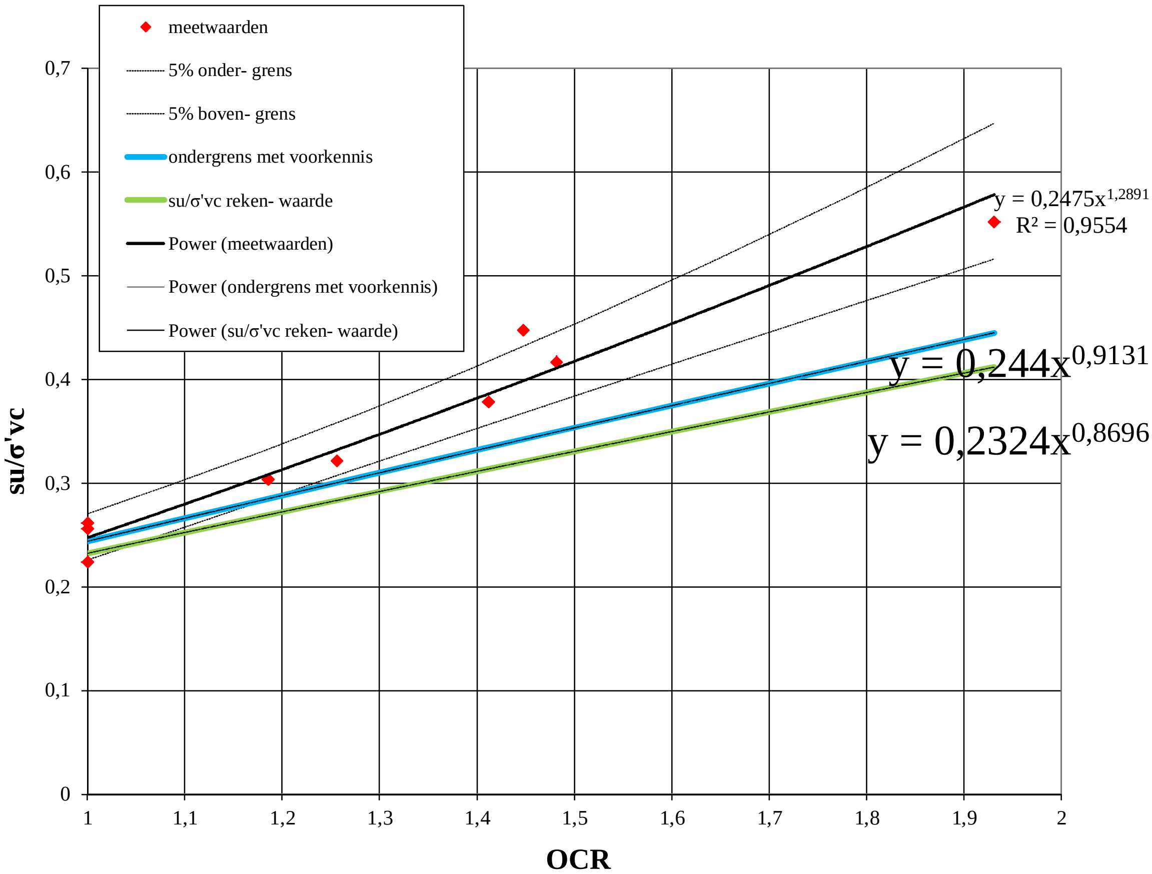
Een alternatieve aanpak zou kunnen zijn om de ongedraineerde schuifsterkte s_u uit te zetten tegen de consolidatiespanning σ'_{vc}. Deze uitwerking is vergelijkbaar met de uitwerking voor cohesie en hoek van inwendige wrijving (zie paragraaf 2.1). Dat zou een nettere uitwerking zijn, omdat dit beide gemeten waarden zijn uit dezelfde proef. Dit geeft minder onzekerheid als gevolg van de gevolgde procedure. Moeilijkheid hierbij is echter dat de relatie tussen σ'_{vc} en s_u wordt bepaald door drie parameters: normaal geconsolideerde ongedraineerde schuifsterkte ratio S, overconsolidatieratio OCR en sterkte toename exponent m. Daarbij zijn dus bij een waarde van σ'_{vc} meerdere waarden van s_u mogelijk, met name omdat OCR erg variabel is. De parameter m zou hierin met voorkennis uit samendrukkingsproeven of CRS proeven kunnen worden ingebracht. Vervolgens zijn er dan nog twee onbekenden: S en OCR. Omdat OCR erg variabel is, is de band van s_u tegen σ'_{vc} erg breed. We moeten dan iets zeggen over de verwachtingswaarde en standaardafwijking van S uitgaande van een verwachtingswaarde van OCR en een grote standaardafwijking van OCR. Deze uitwerking lijkt al met al niet werkbaar.

Vanwege het ontbreken van een logischer alternatief is daarom de aanpak met s_u/σ'_{vc} tegen OCR verder uitgewerkt.

Het spreadsheet berekent verwachtingswaarden, karakteristieke waarden en rekenwaarden van de normaal geconsolideerde ongedraineerde schuifsterkte ratio S en de sterkte toename exponent m op basis van een serie proefresultaten uit triaxiaalproeven of direct simple shear proeven en samendrukkingsproeven of CRS proeven. Hiervoor wordt uitgegaan van de relaties die worden beschreven in hoofdstuk 3. De invoerwaarden uit de triaxiaalproeven en de direct simple shear proeven zijn de ongedraineerde schuifsterkte ratio s_u/σ'_{vc} (-). Uit samendrukkingsproeven of CRS proeven wordt de overconsolidatieratio OCR (-) bepaald. Van beide parameters wordt de logaritme genomen en vervolgens wordt de kleinste kwadratenmethode toegepast conform hoofdstuk 3.

Bij het uitwerken van de proevenverzameling kan in het spreadsheet onderscheid worden gemaakt tussen een lokale of regionale proevenverzameling (zie paragraaf 2.1). Voor een lokale proevenverzameling is α gelijk aan 1,0 en voor een regionale proevenverzameling is α gelijk aan 0,75.

Vaak geeft het afleiden van de exponent m uit samendrukkingsproeven en CRS proeven betrouwbaarder waarden van m dan het fitten van m via lineaire regressie op basis van meetwaarden van s_u/σ'_{vc} versus OCR. Betrouwbaarder wil in dit verband zeggen dat m een kleinere bandbreedte heeft en ook beter aansluit bij waarden uit de internationale literatuur (m moet kleiner zijn dan 1,0 en groter dan 0,6 à 0,7). Daarom kan in het spreadsheet voorkennis over m uit samendrukkingsproeven of CRS proeven worden gebruikt. Daarvoor worden de verwachtingswaarde en standaardafwijking van m in het spreadsheet ingevuld.



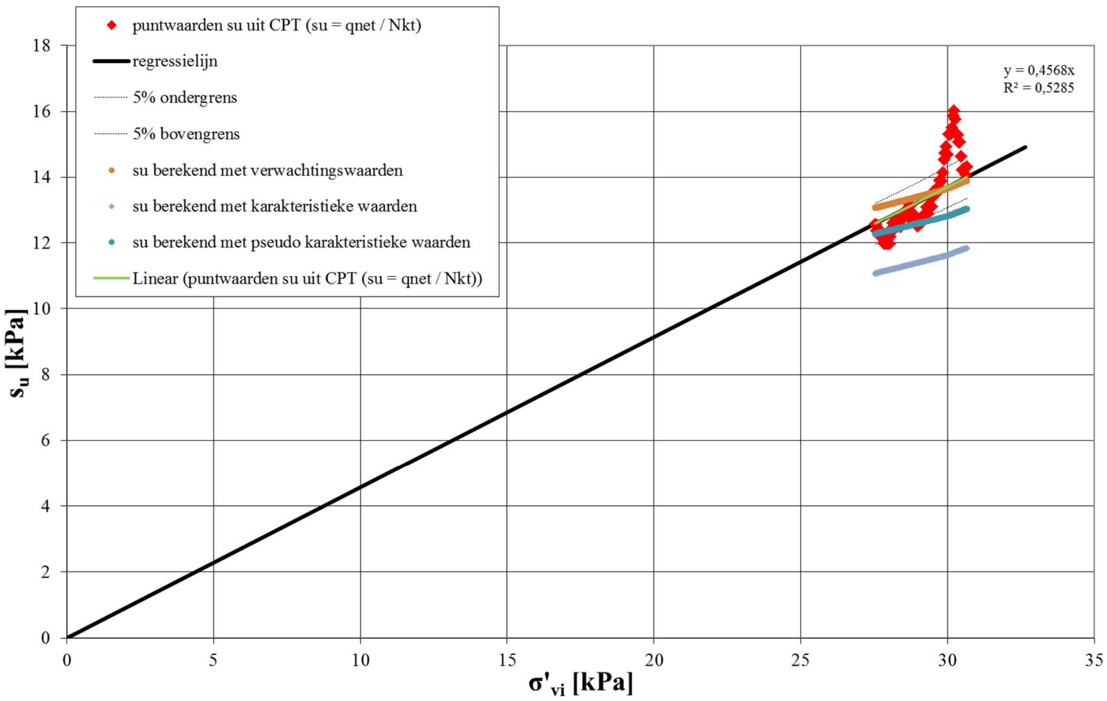
Figuur 2.2 Berekening van verwachtingswaarden, karakteristieke waarden en rekenwaarden van de normaal geconsolideerde ongedraineerde schuifsterkte ratio S en de sterkte toename exponent m.

Wanneer m uit de samendrukkingsproeven of CRS proeven duidelijk afwijkt van de met het spreadsheet gefitte waarde van m op basis van de resultaten uit de triaxiaalproeven of direct simple shear proeven is de met het spreadsheet berekende waarde van de correlatiecoëfficiënt ρ niet meer juist. In dat geval wordt ρ gelijk aan 0 genomen.

In Figuur 2.2 staat de uitwerking voor het bepalen van S en m op basis van s_u/σ'_{vc} tegen OCR weergegeven. In het spreadsheet wordt eerst lineaire regressie op de meetwaarden uitgevoerd en wordt de ondergrens met 5% onderschrijdingskans van s_u/σ'_{vc} tegen OCR berekend. De voorkennis over m uit samendrukkingsproeven of CRS proeven is toegevoegd om de uitwerking bij te kunnen sturen voor onrealistische waarden van m. Door de verwachtingswaarde en standaardafwijking van S te variëren is een tweede ondergrens van

s_u/σ'_{vc} tegen OCR bepaald (ondergrens met voorkennis), die zo dicht mogelijk bij de ondergrens van s_u/σ'_{vc} tegen OCR komt, die met de kleinste kwadratenmethode wordt berekend. De waarden van S zijn zo gekozen dat de ondergrens met voorkennis van s_u/σ'_{vc} tegen OCR vooral goed is voor waarden van OCR die in het veld voorkomen. Op deze manier is het mogelijk om reële verwachtingswaarden en standaardafwijkingen van S en m te vinden, die goed aansluiten bij literatuur waarden.

Verder is belangrijk dat er een goede samenhang is tussen alle parameters. Daarom is een controle uitgevoerd, waarin wordt gecontroleerd of de karakteristieke waarden van S, OCR en m goed aansluiten bij de karakteristieke waarde van s_u uit de sondering (als de materiaalfactoren bekend zijn, kunnen we dat met rekenwaarden doen).



Figuur 2.3 Controle van de samenhang tussen de karakteristieke waarden van S, OCR en m uit de laboratoriumproeven en de karakteristieke waarde van s_u uit de sondering.

In Figuur 2.3 is de controle van de samenhang tussen de karakteristieke waarden van S, OCR en m uit de laboratoriumproeven en de karakteristieke waarde van s_u uit de sondering weergegeven. De sondering geeft immers lokale informatie over s_u ($s_u = q_{net} / N_{kt}$). Hierin zijn q_{net} de gecorrigeerde sondeerweerstand en N_{kt} de empirisch bepaalde conusfactor op basis van een vergelijking tussen sondeerweerstanden en schuifsterktewaarden uit laboratoriumproeven.

Voor de controle is de sondeerweerstand omgewerkt naar de ongedraineerde schuifsterkte s_u, OCR en grensspanning σ'_{vy}. N_{kt} is bepaald door de grensspanning uit de CPT zo netjes mogelijk op de grensspanning σ'_{vy} uit een aantal samendrukkingsproeven en CRS proeven te krijgen. Daarbij zijn de verwachtingswaarden van S en m uit Figuur 2.2 gebruikt (om van ongedraineerde schuifsterkte s_u naar OCR en grensspanning σ'_{vy} te komen). De effectieve spanning en de bijbehorende puntwaarden van de ongedraineerde schuifsterkte uit de CPT zijn hiermee bepaald (dat wil zeggen een stukje van 0,5 m van Gorkum zwaar klei).

Met de kleinste kwadratenmethode is lineaire regressie toegepast op de verwachtingswaarden van s_u uit de sondering. Tevens is de ondergrenswaarde van s_u uit de sondering berekend. Ook hier is er weer de mogelijkheid om de lineaire regressie zonodig bij te sturen. Verder zijn drie s_u-profielen berekend met de verwachtingswaarden, karakteristieke waarden en “pseudo karakteristieke waarden” van S, OCR en m. Hiermee is onderzocht of de verwachtingswaarden van S, grensspanning en m samen tot een verwachtingswaarde van s_u leiden, die past bij de verwachtingswaarde van s_u uit de CPT. Voor de karakteristieke waarden is nagegaan of de karakteristieke waarden van S, grensspanning en m samen tot een karakteristieke waarde van s_u leiden die past bij de karakteristieke waarde van s_u uit de CPT.

Uit de controle blijkt dat de karakteristieke waarden van S, OCR en m tot een lagere berekende waarde van s_u leiden dan de karakteristieke waarde van s_u uit de sondering. Dit kan te maken hebben met het verschil in het aantal waarnemingen (heel veel voor s_u en OCR en veel minder voor S) of met het feit dat de berekende s_u is gebaseerd op de combinatie van drie parameters waarvoor drie keer de karakteristieke ondergrenswaarde is toegepast. Dit laatste is wellicht wat te conservatief. Dit zou kunnen worden opgelost met een soort “pseudo karakteristieke waarde” aanpak. Daarbij wordt niet voor alle parameters de karakteristieke ondergrenswaarde toegepast, maar voor de meest dominante parameter wordt de karakteristieke waarde toegepast en voor de overige parameters een hogere waarde. Het principe van “pseudo karakteristieke waarden” wordt toegelicht in Bijlage 1 van het Technisch Rapport Waterkerende Grondconstructies (TAW, 2001).

Het is van belang dat de karakteristieke waarden van S, OCR en m samen met de effectieve spanning in een stabiliteitsberekening zoveel mogelijk hetzelfde s_u-profiel geven als de sondering. Op het moment dat dit het geval is, kan de statistische uitwerking voor het bepalen van S en m op basis van s_u/σ'_{vc} tegen OCR als voldoende accuraat worden gezien (hoewel het vanuit de wiskunde / statistiek niet volledig correct is, vanwege de onzekerheid van OCR (uitgaande van toepassing van de recompression of “Norwegian style” procedure), die niet in rekening wordt gebracht met de gekozen uitwerking met de kleinste kwadratenmethode).

3. Achtergrond van in de spreadsheets toegepaste regressieanalyse

3.1 *Kleinste kwadratenmethode voor enkelvoudige lineaire regressie*

Bij (enkelvoudige) lineaire regressieanalyse wordt een trendfunctie:

$$t = \hat{a}_1 + \hat{a}_2 \, s \tag{3.1}$$

bepaald, die zo goed mogelijk past bij een set waarnemingen $\{T_i\ ,\ s_i\}$ ($i=1 \ldots n$). De T_i zijn gemeten waarden van een of andere variabele, die (mede) afhankelijk zijn van ingestelde waarden s_i. De regressieparameters $\hat{a}_1$ en $\hat{a}_2$ worden bepaald volgens de kleinste kwadratenmethode. Dit houdt in dat deze parameters zodanig worden bepaald dat de som van de kwadraten van de residuen $(T_i - \hat{a}_1 - \hat{a}_2\, s_i)$ minimaal is, dus:

$$R^2 = \sum_{i=1}^{n} (T_i - \hat{a}_1 - \hat{a}_2 \, s_i)^2 \tag{3.2}$$

is minimaal. Dit leidt tot de volgende vergelijkingen waaraan $\hat{a}_1$ en $\hat{a}_2$ moeten voldoen:

$$\frac{\partial R^2}{\partial \hat{a}_1} = -\sum_{i=1}^{n} 2(T_i - \hat{a}_1 - \hat{a}_2 \, s_i) = 0 \quad => \quad n\hat{a}_1 + (\sum_{i=1}^{n} s_i)\hat{a}_2 = \sum_{i=1}^{n} T_i \tag{3.3}$$

$$\frac{\partial R^2}{\partial \hat{a}_2} = -\sum_{i=1}^{n} 2(T_i - \hat{a}_1 - \hat{a}_2 \, s_i)s_i = 0 \quad => \quad (\sum_{i=1}^{n} s_i)\hat{a}_1 + (\sum_{i=1}^{n} s_i^2)\hat{a}_2 = \sum_{i=1}^{n} T_i \, s_i \tag{3.4}$$

De oplossing luidt:

$$\hat{a}_2 = \frac{\sum_{i=1}^{n} T_i \, (s_i - \bar{s})}{\sum_{i=1}^{n} (s_i - \bar{s})^2} \qquad \text{hierin is} \qquad \bar{s} = (\sum_{i=1}^{n} s_i) / n \tag{3.5}$$

en:

$$\hat{a}_1 = \frac{\sum_{i=1}^{n} (T_i - \hat{a}_2 \, s_i)}{n} \tag{3.6}$$

De minimale R^2 wordt gevonden door deze oplossing te gebruiken in vgl. (3.2).

3.2 *Onderliggend stochastisch model*

De in paragraaf 3.1 beschreven methode wordt vaak toegepast als puur numerieke techniek zonder associatie met het onderliggend stochastisch model. Het onderliggende stochastische model is dat *t* een normaal verdeelde stochastische variabele is, die lineair afhankelijk is van *s.* Of de aanname dat *t* een normaal verdeelde stochastische variabele is terecht is voor toepassing bij schuifsterkteparameters, moet formeel aan de hand van statistische toetsen op beschikbare gegevens over schuifsterkteparameters worden vastgesteld. De verwachtingswaarde van t is:

$$E[t] = a_1 + a_2 \, s \tag{3.7}$$

en met standaardafwijking σ_t. De stochastische variabele *t* en waargenomen realisaties T_i ervan we kunnen derhalve geschreven worden als:

$$t = a_1 + a_2 \, s + \varepsilon \quad \Rightarrow \quad T_i = a_1 + a_2 \, s_i + \varepsilon_i \tag{3.8}$$

Hierin is ε een normaal verdeelde stochastische variabele met verwachtingswaarde gelijk 0 en standaardafwijking gelijk σ_t en zijn de ε_i realisaties van ε in de steekproef $\{T_i, s_i\}$ ($i=1$ … n). De aanname hieronder is dat de realisaties ε_i en ε_j onafhankelijk zijn. Dit is geen noodzakelijke aanname, maar doorgaans wordt hieraan voldaan in steekproeven.

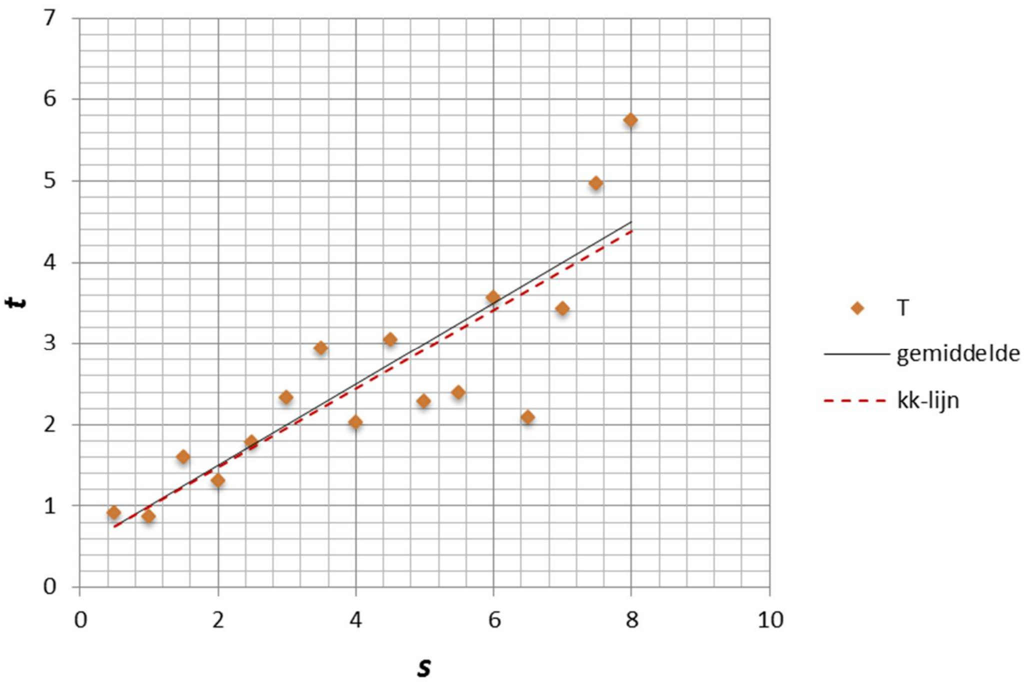
De model parameters a_1 en a_2 en σ_t zijn doorgaans onbekenden, die aan de hand van de steekproef van waarnemingen $\{T_i, s_i\}$ ($i=1 \ldots n$) moeten worden bepaald. Er geldt dat:

- De $\hat{a}_1$ en $\hat{a}_2$ volgens vgl. (3.6) en (3.5) zijn op te vatten als (realisaties van) stochastische variabelen. Immers ze zijn afhankelijk van de stochastische variabele t (via de realisaties T_i). Berekende waarden van $\hat{a}_1$ en $\hat{a}_2$ zijn zuivere schatters zijn van a_1 en a_2, met andere woorden:

$$E[\hat{a}_1] = a_1 \text{ en } E[\hat{a}_2] = a_2 \tag{3.9}$$

- $R^2/(n-2)$ (en deze grootheid noteren we verder als S_t^2) een zuivere schatter van de variantie σ_t^2 is. Met andere woorden: $E[S_t^2] = \sigma_t^2$. In het vervolg gaan we er van uit dat S_t^2 een (voldoend) nauwkeurige schatting van σ_t^2 levert. Theoretisch statistisch gezien valt daar uiteraard op af te dingen, maar voor de praktische toepassing hier is dat acceptabel.

Figuur 3.1 geeft een illustratie van het model. De lijn “gemiddelde” (ononderbroken zwarte lijn) is de lijn volgens vgl. (3.7). Hierin is $a_1=0,5$ en $a_2=0,5$. De rode ruitjes geven (gegenereerde) realisaties van T_i, volgens vgl. (3.8). Hierbij is uitgegaan van een normale kansverdeling van ε, met verwachtingswaarde 0 en variantie $0,5$. De rode streepjeslijn in de figuur is de berekende kleinste kwadraten lijn $t = \hat{a}_1 + \hat{a}_2 s$, waarbij $\hat{a}_1$ en $\hat{a}_2$ berekend zijn o.b.v. de gegenereerde realisaties (T_i,s_i) met behulp van de vergelijkingen (3.6) en (3.5). In dit geval lijkt ligt de benadering (rode streepjeslijn) dicht bij de oorspronkelijke lijn (zwarte lijn) waarvan uitgegaan bij het genereren van de set “pseudo” realisaties. Hier komen we in paragraaf 3.3 nader op terug.



Figuur 3.1 Illustratie van model.

De varianties van $\hat{a}_1$ en $\hat{a}_2$ worden als volgt berekend:

$$\sigma^2(\hat{a}_1) = S_t^2\left(\frac{1}{n}\left(1+\frac{\sum_{i=1}^{n} s_i^2}{n\sum_{i=1}^{n}(s_i-\bar{s})^2}\right)\right) \tag{3.10}$$

$$\sigma^2(\hat{a}_2) = \frac{S_t^2}{\sum_{i=1}^{n}(s_i-\bar{s})^2} \tag{3.11}$$

De schatters $\hat{a}_1$ en $\hat{a}_2$ zijn onderling afhankelijk. De covariantie is:

$$cov(\hat{a}_1,\hat{a}_2) = E[(\hat{a}_1-a_1)(\hat{a}_2-a_2)] = -\frac{\bar{s}}{\sum_{i=1}^{n}(s_i-\bar{s})^2}S_t^2 \tag{3.12}$$

en de correlatie is (per definitie):

$$\rho(\hat{a}_1,\hat{a}_2) = \frac{cov(\hat{a}_1,\hat{a}_2)}{\sigma(\hat{a}_1)\,\sigma(\hat{a}_2)} \tag{3.13}$$

In appendix 1 zijn wiskundige afleidingen van de vergelijkingen (3.10) … (3.12) gegeven.

3.3 *Karakteristieke schattingen*

Aan de hand de schatters $\hat{a}_1$, $\hat{a}_2$, de varianties ervan en de correlatie, kunnen karakteristieke schattingen (d.w.z. onder- en bovengrenzen met 5% kans op over- of onderschrijding; betrouwbaarheidsinterval 90%). Voor de puntwaarden zijn die grenzen:

$$t_{kar} = \hat{a}_1 + \hat{a}_2\, s \pm t_{n-2}^{0.95}\sqrt{\sigma^2(\hat{a}_1)+s^2\sigma^2(\hat{a}_2)+2\rho(\hat{a}_1,\hat{a}_2)\,s\sigma(\hat{a}_1)\sigma(\hat{a}_2)+S_t^2} \tag{3.14}$$

Hierin is $t_{n-2}^{0.95}$ de Student-t met 95% onderschrijdingskans. De “+” van het “±” in vgl. (3.14) geeft de karakteristieke bovengrens en “-“ de karakteristieke ondergrens. De eerste drie termen onder het wortelteken representeren de variantie van de onzekerheid van het gemiddelde van *t*, de laatste term de variantie van de ruimtelijke spreiding van *t*. Voor karakteristieke waarden van “gemiddelden” van *t,* aangegeven als (zie in hoofdstuk 3 bijvoorbeeld) grondlaaggemiddelden is er een bijna identieke uitdrukking:

$$\bar{t}_{kar} = \hat{a}_1 + \hat{a}_2\, s \pm t_{n-2}^{0.95}\sqrt{\sigma^2(\hat{a}_1)+s^2\sigma^2(\hat{a}_2)+2\rho(\hat{a}_1,\hat{a}_2)\,s\sigma(\hat{a}_1)\sigma(\hat{a}_2)+(1-\alpha)S_t^2} \tag{3.15}$$

Het verschil zit in de mate waarin de ruimtelijke spreiding in rekening wordt gebracht. De factor (*1-α*) in vgl. (3.15) is een reductie factor. Deze is afhankelijk van het (stochastische) ruimtelijke model waarmee de spreiding van *t* te beschrijven is. In dat model speelt de verhouding tussen de variantie van *t* op een locatie en de regionale variantie een rol. Die verhouding wordt weergegeven met *α*. De variantie van t op een locatie is de variantie van de spreiding van "puntwaarden" van *t* ten opzichte van een lokaal gemiddelde. De regionale variantie is de variantie van de spreiding van "puntwaarden" van *t* t.o.v. een regionaal gemiddelde. Geven we de regionale variantie aan met σ_t^2 dan is de variantie van *t* op een locatie gelijk aan $\alpha\,\sigma_t^2$. Bij gevolg is de variantie van de spreiding van lokale gemiddelden t.o.v. het regionale gemiddelde $(1-\alpha)\sigma_t^2$. Dit model voor het statistisch beschrijven van ruimtelijke variatie is o.a. beschreven in (Calle, 2007 en 2008). De verhouding tussen lokale en regionale variantie wordt hierin aangeduid met het symbool "α", in eerdere publicaties (Calle, 1996) met "a" en in de TAW-leidraad voor het ontwerpen van rivierdijken (deel 2) (TAW, 1989) met ($1\text{-}\Gamma^2$). De wisselende notaties reflecteren het voortschrijdend inzicht door de jaren heen. De term $(1-\alpha)\,S_t^2$ in vgl. (3.15) reflecteert dat bij de bepaling van de karakteristieke schatting van de gemiddelde waarde van *t*, op een willekeurige locatie binnen een gebied waarvoor een regionale proevenverzameling is opgesteld, ermee rekening gehouden moet worden dat deze lokale gemiddelden variëren ten opzichte van het regionale gemiddelde. De overige termen in vgl. (3.15) reflecteren de statistische onzekerheid van schattingen van het regionale gemiddelde.

In het oorspronkelijke spreadsheet is een van vgl. (3.15) afwijkende formule gebruikt, die (m.m.) luidt:

$$\bar{t}_{kar} = \hat{a}_1 + \hat{a}_2\,s \pm t_{n-2}^{0.95}\sqrt{(\sigma^2(\hat{a}_1) + s^2\sigma^2(\hat{a}_2) + 2\rho(\hat{a}_1,\hat{a}_2)\,s\,\sigma(\hat{a}_1)\sigma(\hat{a}_2))\,(n(1-\alpha)+1)} \tag{3.16}$$

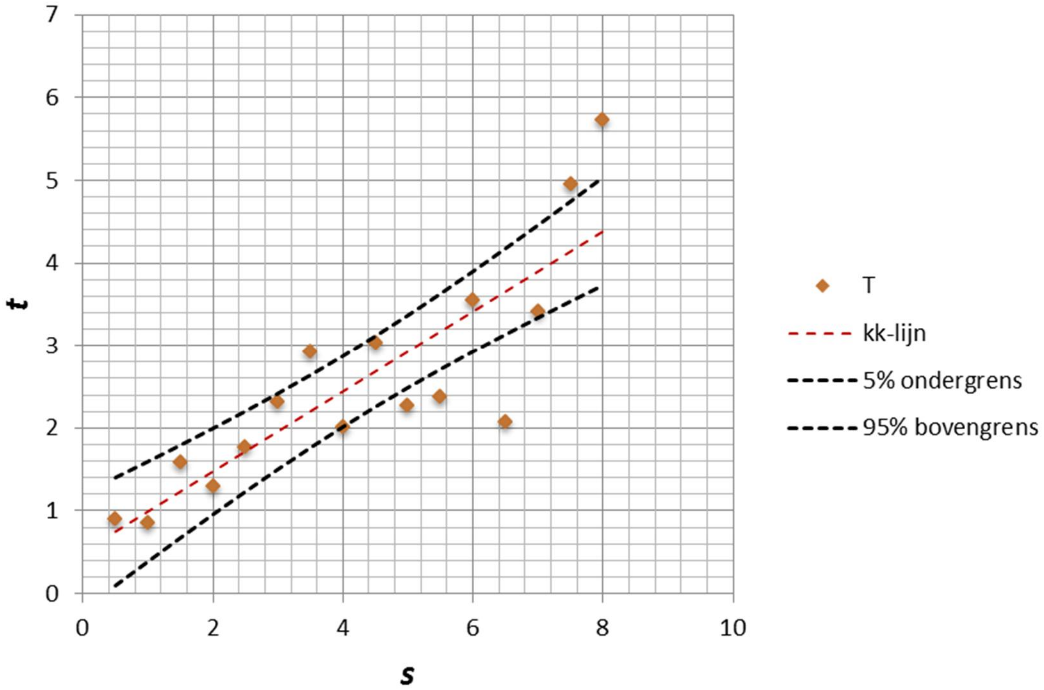
De formules (3.15) en (3.16) zouden overeenkomen als:

$$(\sigma^2(\hat{a}_1) + s^2\sigma^2(\hat{a}_2) + 2\rho(\hat{a}_1,\hat{a}_2)\,s\,\sigma(\hat{a}_1)\sigma(\hat{a}_2)) = \frac{S_t^2}{n} \tag{3.17}$$

Dit is alleen het geval wanneer de stochast *t* een enkelvoudige stochast is, dus niet afhankelijk van de parameter *s*. De in het oorspronkelijke spreadsheet opgenomen relaties volgens vgl. (3.16) voor het bepalen van karakteristieke schattingen, moeten derhalve vervangen worden door relaties volgens vgl. (3.15).

In Figuur 3.2 zijn de 5% onder- en de 95% bovengrens weergegeven die berekend zijn met vgl. (3.15) (met daarin *α=1*). De onder- en bovengrens markeren de ruimte waarbinnen de rechte "gemiddelde waarde"-lijn kan liggen (met 90% betrouwbaarheid). Elke rechte lijn die tussen de onder- en bovengrenzen ligt is in principe mogelijk (maar ze zijn uiteraard niet alle even waarschijnlijk). Hieruit blijkt dat het dicht bij elkaar liggen van de "gemiddelde lijn" en de "kleinste kwadratenlijn" in Figuur 3.1 een beetje op toeval berust. Let op, de lijnen markeren dus niet het 90% betrouwbaarheidsgebied waarbinnen de punten (T_i,s_i) moeten liggen.

In het geval de t-waarde per definitie positief moet zijn (denk aan een schuifsterkteparameter), dan zullen het gemiddelde en de ondergrens van *t* voor elke waarde van *s* positief moeten zijn.



Figuur 3.2 Gebied waarbinnen de gemiddelde waarden lijn kan liggen (90% betrouwbaarheid).

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Appendix 1: Wiskundige afleidingen

Te bewijzen: $\hat{a}_1$ en $\hat{a}_2$ zijn zuivere schatters van a_1 en a_2, m.a.w. $E[\hat{a}_1] = a_1$ en $E[\hat{a}_2] = a_2$:
Substitutie van de $T_i = a_1 + a_2 s_i + \varepsilon_i$ (volgens het stochastisch model volgens vgl. (3.8)) in vgl. (3.5) levert:

$$\hat{a}_2 = \frac{\sum_{i=1}^{n} T_i (s_i - \bar{s})}{\sum_{i=1}^{n} (s_i - \bar{s})^2} \quad ==> \quad = \frac{\sum_{i=1}^{n} (a_1 + a_2 s_i + \varepsilon_i)(s_i - \bar{s})}{\sum_{i=1}^{n} (s_i - \bar{s})^2}$$

$$= \frac{\sum_{i=1}^{n} (a_1 (s_i - \bar{s}) + a_2 (s_i - \bar{s})(s_i - \bar{s}) + a_2 \bar{s} (s_i - \bar{s}) + \varepsilon_i (s_i - \bar{s}))}{\sum_{i=1}^{n} (s_i - \bar{s})^2} \qquad \text{(A1.1)}$$

$$= \frac{a_2 \sum_{i=1}^{n} (s_i - \bar{s})^2 + \sum_{i=1}^{n} \varepsilon_i (s_i - \bar{s})}{\sum_{i=1}^{n} (s_i - \bar{s})^2} = a_2 + \frac{\sum_{i=1}^{n} \varepsilon_i (s_i - \bar{s})}{\sum_{i=1}^{n} (s_i - \bar{s})^2}$$

Omdat $E[\varepsilon_i] = 0$ is dus $E[\hat{a}_2] = a_2$ (qed).
Analoog geldt, uitgaande van vgl. (3.6) :

$$\hat{a}_1 = \frac{\sum_{i=1}^{n} (T_i - \hat{a}_2 s_i)}{n} \quad ==> \quad = \frac{\sum_{i=1}^{n} (a_1 + a_2 s_i + \varepsilon_i - \hat{a}_2 s_i)}{n} \qquad \text{(A1.2)}$$

$$= a_1 + (a_2 - \hat{a}_2) \frac{1}{n} \sum_{i=1}^{n} s_i + \frac{1}{n} \sum_{i=1}^{n} \varepsilon_i$$

En omdat $E[\varepsilon_i] = 0$ en $E[\hat{a}_2 - a_2] = 0$ is derhalve $E[\hat{a}_1] = a_1$ (qed).
De varianties van $\hat{a}_1$ en $\hat{a}_2$ worden als volgt berekend:

$$\sigma^2(\hat{a}_2) = E[(\hat{a}_2 - a_2)^2] = E[\left(\frac{\sum_{i=1}^{n}\varepsilon_i\,(s_i - \bar{s})}{\sum_{i=1}^{n}(s_i - \bar{s})^2}\right)^2]$$

$$= \frac{1}{\left(\sum_{i=1}^{n}(s_i - \bar{s})^2\right)^2}\sum_{i=1}^{n}\sum_{j=1}^{n}(E[\varepsilon_i\varepsilon_j]\,(s_i - \bar{s})(s_j - \bar{s})) \qquad \text{(A1.3)}$$

$$= \frac{\sigma_t^2}{\sum_{i=1}^{n}(s_i - \bar{s})^2}$$

Omdat $E[\varepsilon_i\varepsilon_j] = 0$ als *j≠i* en $E[\varepsilon_i\varepsilon_i] = \sigma_t^2 \quad (i = 1 \ldots n)$. Verder geldt:

$$\sigma^2(\hat{a}_1) = E[(\hat{a}_1 - a_1)^2]$$

$$=> vgl(\text{A1.2}) \quad = E[\left((a_2 - \hat{a}_2)\frac{1}{n}\sum_{i=1}^{n}s_i + \frac{1}{n}\sum_{i=1}^{n}\varepsilon_i\right)^2]$$

$$= \sigma^2(\hat{a}_2)\,\bar{s}^2 + \frac{\sigma_t^2}{n} = \bar{s}^2\frac{\sigma_t^2}{\sum_{i=1}^{n}(s_i - \bar{s})^2} + \frac{\sigma_t^2}{n} \qquad \text{(A1.4)}$$

$$= \frac{\sigma_t^2}{n}\left(1 + \frac{(\sum_{i=1}^{n}s_i)^2}{n\sum_{i=1}^{n}(s_i - \bar{s})^2}\right)$$

De uitdrukkingen zijn zodanig gekozen dat ze overeenkomen met de notaties in de spreadsheet en (Van Esch, 1997).
De covariantie van $\hat{a}_1$ en $\hat{a}_2$ wordt berekend als:

$$cov(\hat{a}_1, \hat{a}_2) = E[(\hat{a}_1 - a_1)\,(\hat{a}_2 - a_2)] = -\bar{s}\;E[(\hat{a}_2 - a_2)^2]$$

$$= -\bar{s}\;\sigma^2(\hat{a}_2) = -\frac{\bar{s}}{\sum_{i-1}^{n}(s_i - \bar{s})^2}\sigma_t^2 \qquad \text{(A1.5)}$$

Hierin is gebruik gemaakt van de relatie in vgl. (3.6):

$$\hat{a}_1 = \frac{\sum_{i=1}^{n}(T_i - \hat{a}_2\, s_i)}{n} = a_1 + a_2\bar{s} - \hat{a}_2\bar{s} + \frac{1}{n}\sum_{i=1}^{n}\varepsilon_i$$

$$=> \quad \hat{a}_1 - a_1 = -(\hat{a}_2 - a_2)\,\bar{s} + \frac{1}{n}\sum_{i=1}^{n}\varepsilon_i \qquad \text{(A1.6)}$$

$$=> \quad (\hat{a}_1 - a_1)(\hat{a}_2 - a_2) = -(\hat{a}_2 - a_2)^2\,\bar{s} + (\hat{a}_2 - a_2)\frac{1}{n}\sum_{i=1}^{n}\varepsilon_i$$

$$=> \quad E[(\hat{a}_1 - a_1)(\hat{a}_2 - a_2)] = -\bar{s}\; E[(\hat{a}_2 - a_2)^2] + E[(\hat{a}_2 - a_2)\frac{1}{n}\sum_{i=1}^{n}\varepsilon_i]$$

We beschouwen de tweede term van het rechterlid apart en substitueren hierin voor $(\hat{a}_2 - a_2)$ de uitdrukking in het tweede rechterlid van vgl. (A1.3), dan volgt:

$$E[(\hat{a}_2 - a_2)\frac{1}{n}\sum_{i=1}^{n}\varepsilon_i] = E[\frac{\sum_{k=1}^{n}\varepsilon_k(s_k - \bar{s})}{\sum_{k=1}^{n}(s_k - \bar{s})^2}\,\frac{1}{n}\sum_{i=1}^{n}\varepsilon_i] =$$

$$\frac{\sum_{k=1}^{n}\sum_{i=1}^{n}E[\varepsilon_i\varepsilon_k](s_k - \bar{s})}{n\sum_{k=1}^{n}(s_k - \bar{s})^2} = \frac{\sigma_t^2\sum_{k=1}^{n}(s_k - \bar{s})}{n\sum_{k=1}^{n}(s_k - \bar{s})^2} = 0 \qquad \text{(A1.7)}$$

Omdat $E[\varepsilon_i\varepsilon_k] = 0$ als k≠*i* en $E[\varepsilon_k\varepsilon_k] = \sigma_t^2 \quad (i = 1 ... n)$. Daarmee is de vgl. (A1.5) bewezen.

Bijlage 2: Rapport 'Characteristic values of soil properties in Dutch codes of practice' (Deltares)

https://www.deltares.nl/expertise/publicaties/characteristic-values-of-soil-properties-in-dutch-codes-of-practice-theoretical-backgrounds-and-assumptions

Deltares

Characteristic values of soil properties in Dutch codes of practice

Theoretical backgrounds and assumptions

Characteristic values of soil properties in Dutch codes of practice
Theoretical backgrounds and assumptions

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Summary

Soil properties usually show significant uncertainty due to a combination of spatial variability, limited availability of measurements and other sources of uncertainty. Most modern design codes are based on Load and Resistance Factor Design. In these codes, uncertainties in soil properties are covered by design values of geotechnical parameters, which are a combination of characteristic values, often defined as cautious estimates of properties affecting the limit state, and partial factors. Characteristic values of geotechnical parameters are determined either based on engineering judgement or on a stochastic/statistical basis, of which the latter is subject of this paper. This report elaborates on the theoretical backgrounds and the practical assumptions made for the derivation of characteristic values based on a statistical basis. The methods were initially derived for Dutch flood defenses but are generically applicable to geotechnical design and assessment.

The following aspects relevant to characteristic values are covered:

- The overall semi-probabilistic framework and the role of characteristic values therein;
- The modeling of spatial variability of a soil property, including the difference between random spatial models based on local datasets and regional datasets, the latter usually being a merge of local datasets within a large region. It will be demonstrated that these two spatial model approaches, though they look quite similar, may lead to substantial different characteristic value assessments.
- "Variance reduction", i.e. the effect of averaging of spatial variability, involved in the assessment of ä soil property based parameter for geotechnical analyses;
- The derivation of characteristic values of geotechnical parameters, including the effect of adopted spatial model (based on local or regional data), "variance reduction", statistical uncertainty, measurement errors and correlated versus uncorrelated data.

Though a stochastic/statistical approach inherently suggests objectivity, some of the model parameters of the adopted random field model inevitably still require engineering intuition based assessment. Actual available data from soil investigation is most often too limited for meaningful statistical inference of model parameters, other than expected mean values or standard deviations. Assessments of other model parameters should then be based on engineering interpretation of the available data, focusing on cautious but reasonable assumptions for the assessment of characteristic geotechnical parameters.

We believe that for any improvement or alternative proposals to operationally define characteristic values, it is crucial to address all these aspects. Of course, that does not mean that all aspects need be ultimately part of the recipe. As we show, also the current formulations work with simplifications based on (mostly judgement-based) assumptions. Yet is seems important to us that all essential elements are covered in the justification.

Contents

1 Introduction

1.1 Rationale and scope

Parameters in a geotechnical analysis, reflecting soil (strength or load) properties, are usually subject to substantial uncertainty. Unlike man-made materials such as concrete and steel, soils have a much higher variability of material properties as it is a natural material influenced by geologic processes. Although investments to map ground conditions have proven valuable returns, typically limited ground and soil investigation data are available to quantify soil properties and map heterogeneities. Uncertainties must be taken into account, to safely design and assess geotechnical structures.

Historically speaking, the consideration of safety in geotechnical designs, and nowadays increasingly in the assessment of existing structures, has evolved from using overall factors of safety based on mean property values to more sophisticated load and resistance factor design (LRFD), which became mainstream in structural engineering in the 1980’s. The overall factors of safety were mostly based on empirical experience and engineering judgement. With the introduction of LRFD, reliability-based concepts became applicable to define design values and characteristic values of load and resistance properties in probabilistic terms.

The authors recognize confusion about some concepts and assumptions underlying the approach to characteristic values of geotechnical parameters in Dutch guidelines and codes of practice due to lack of background documentation. The objective of this report is to elaborate on the reliability backgrounds of the design values of geotechnical parameters, and specifically on the definition of characteristic values in probabilistic terms, as well as the derivation of widely used equations for this.

The report mostly focuses on the probabilistic and statistical elements in the determination of characteristic values as implemented in Dutch guidelines for flood defenses (see e.g. TAW, 1989 and its successors,) and Dutch codes of practice in the National Annex to the EuroCode (NEN-EN-1997). This focus is chosen as most of the described theoretical concepts in this report has been developed within the context of Dutch flood defenses., further referred to as dikes in this paper. However, the theory described is generically applicable to other geotechnical structures and line-infrastructure as well. Except for the application of regional soil models (see Chapter 3), many aspects are covered in Eurocode 7 (EN 1997-1:2004) as well.

The report is intended for practitioners and researchers who want to know the background behind current methods and equations for the determination of characteristic values. This will enable them to interpret the back ground of current approach correctly and to make unbiased comparisons with the status quo when developing new methods.

1.2 Characteristic values

According Eurocode 7 (clause 2.4.5.2 from EN 1997-1:2004), the characteristic value of a geotechnical parameter shall be selected as a cautious estimate of the value affecting the occurrence of the limit state. Herein, we take the statistical approach in this report such that calculated probability of a worse value is not greater than 5%. This report deals with how to determine these characteristic values.

The definition of characteristic values is closely related to the required reliability targets in the Eurocodes and in the safety standards for flood defenses through the semi-probabilistic verification format with partial factors. This implies that the operational definitions of characteristic values always need to be considered in conjunction with the overall safety or reliability concept for limit state verification in a guideline or code of practice.

The following aspects are of importance in order to define and determine characteristic values:

1. The overall framework for semi -probabilistic reliability-based design (i.e. Load and Resistance Factor Design). The concept of characteristic values (load and strength) to be used in combination with partial safety factors was launched during the introduction of probability-based design of civil engineering structures in the 1980's. Choices of definitions of characteristic values of load and strength parameters interdepend with choices of partial safety factors and the combinations relate to required levels of structural safety. Though not the main issue of this report, this framework is a crucial, but an often-ignored notion in discussions on characteristic values.

2. The choice of the (spatial) stochastic model, which reflects natural "random" variability of a soil properties within a soil layer. This is a basic element for probabilistic geotechnical reliability analyses, and thus also for determination of characteristic soil properties for semi probabilistic analyses. (Vanmarcke, 1977) introduced a random field model, which has been a basic assumption in geotechnical reliability analysis since then. It was adopted in several recent publications, aimed to substantiate Eurocode issued definitions of characteristic soil parameter (Schneider & Schneider, 2013; Prästings et al., 2018; Ching et al., 2020) and, with some focus on slope stability of regional dikes in the Netherlands, (Hicks et al., 2019; Varkey 2020). In the course of developing an early version of the Dutch river dike code (TAW, 1989), it was found that this "basic" random field model did not fully match with the available regional test datasets, used for Dutch river dike design. An extension of the model was needed to enable accommodation of the large variation of means of the local test sets, which formed the regional test set.

3. Translation of field data or derived values into characteristic values of geotechnical parameters as used in computation models for the verification limit states: Typical notions, in this respect, concern translation of acquired data at (small volume) measurement scale to parameters representative for potential failure mode volumes, e.g. slope slides. This may involve substantial variance reduction, depending on the dimension scale of failure modes and the type computation model for reliability verification. Also, the role of measurement uncertainty should, and will, be considered.

The second and third aspects form the main part of this report, as they are the basis to derive formulas for the computation of characteristic values of soil properties. Simplification of these formulas are needed, additionally, to arrive at formulas which are tractable for everyday practice as currently implemented in the design and assessment guidelines for flood defenses in the Netherlands and in the Dutch National Annex to the Eurocode 7.

Recently, numerous proposals have been made to operationally define characteristic values, also for conditions deviating from the elaborations in this article (i.e. stationary random fields). For example, soil parameters with depth trends or correlated parameters require different treatment. An overview of these developments can be found in ISSMGE-TC304 (2021). This is outside the scope of this report

The possibility to choose characteristic values based on engineering judgement, as given for example in the Eurocode 7 (NEN-EN-1997, 2004) remains of course open to practitioners, and may even be applied in many instances. We do, however, emphasize that even when statistics or probability concepts are not explicitly used, the same level of confidence should be targeted from a reliability point of view. And, hence, considerations that will be discussed here, should, one way or the other, also be addressed in non-statistical approaches.

As may have been noted, we distinguish in this report between soil properties and geotechnical parameters. A soil property is a (actually measurable) quantity which can be attributed to a (mostly small) volume of soil within a soil unit. A geotechnical parameter, is a quantity to be applied in a computation model, attributable to volumes or surfaces involved in limit state failure. It is derived from soil property data, and it should account for effects like spatial averaging, and uncertainties which may play a role in the translation from measured data to computation model parameter.

1.3 Outline

In order to properly place the concept of characteristic values in the overall reliability framework of the Eurocodes, and of many related Dutch geotechnical guidelines, section 2 of this report outlines the concepts of (target) reliability and design values in LRFD. Section 3 discusses the modelling of spatial variability and uncertainty in soil properties. Subsequently, the crucial concept of spatial averaging is treated in section 4. The definitions and concepts for characteristic values in probabilistic terms are then given in section 5 (with detailed derivations available in the annex).

2 Overall (semi) probabilistic framework

The Eurocodes are based on reliability concepts (JRC, 2021), as are guidelines for design and safety assessment of flood defenses in the Netherlands, i.e. (TAW, 1989), (OI, 2014), (WBI,2017) for primary and (LTV, 2017) for regional flood defenses. This entails that the underlying reliability requirement refers to an acceptable or target probability of failure or target reliability, operationally defined as the probability of exceeding a limit state. The reliability verification can in principal be done fully probabilistically (e.g. Schweckendiek et. al, 2017), but most guidelines describe semi-probabilistic limit state verifications based on design values.

The essential matter with probability-based approaches to characteristic values is relating their degree of confidence to the overall design approach followed in each code of practice including the target reliability and the partial factors. To that end, in this section we will briefly discuss target reliability values and semi-probabilistic verifications with focus on the role of characteristic values therein.

2.1 Target reliabilities

Table 1 shows the target reliability index β_T for ULS (i.e. structural failure) as a function of the consequence class, based on the Dutch National Annex to the Eurocode. The Eurocode itself defines a framework of (three) classes of societal impact of a structural failure (in terms of damage and/or casualties or injured people), at an aggregate level. Namely the consequence classes distinguish between CC1, CC2 and CC3, based on the societal impact of a structural failure (in terms of damage and/or injured/killed people), at an aggregate level.

Table 2.1: Target reliability index and corresponding probability of failure for a reference period of 50 years in Dutch National Annex to EN 1990 for consequence classes CC1, CC2 and CC3

Consequence class	$\boldsymbol{\beta_T}$	$\boldsymbol{P_{f,T}}$	***Societal Impact of Collapse***
CC1	*3.3*	$4.8 \cdot 10^{-4}$	*Low consequence for loss of human life, and economic, social or environmental consequences small or negligible*
CC2	*3.8*	$7.2 \cdot 10^{-5}$	*Medium consequence for loss of human life, economic, social or environmental consequences considerable*
CC3	*4.3*	$8.5 \cdot 10^{-6}$	*High consequence for loss of human life, or economic, social or environmental consequences very great.*
Note 1: The target reliability values refer to individual structural members and/or individual limit states (JRC, 2021). *Note 2: When translating the target reliability index values to reference periods other than 50 years, the correlation between years and deterioration (if applicable) need to be considered.*			

The relation between required reliability index β_T and the corresponding probability of failure $P_{f,T}$ is given by $P_{f,T} = \Phi(-\beta_T)$, where $\Phi(\cdot)$ denotes the standard normal (Gaussian) probability function. Though the target reliability values in Table 2.1 were initially derived for buildings, since stated in EN 1990, they are also considered applicable to geotechnical structures.

Flood defenses in the Netherlands are subject to a different regime of reliability targets, based on societal and economic impacts of flooding. Requirements for primary defenses, often stricter than Table 2.1, are laid down in the guidelines (IO, 2014) and (WBI, 2017), and often less strict than in Table 2.1 for regional flood defenses in (LTV,2017).

2.2 Semi-probabilistic verification

The fundamental basis of a semi-probabilistic verification is that a limit state verification with limit state function $g(X)$ using design values has to meet the condition:

$$g(X_d) \geq 0 \tag{2.1}$$

The X_d are the so-called design values of the random variables. In terms of reliability theory, the design values represent the values of the random variables X for which the probability density at failure, (where $g(X) = 0$, after transformation into the standardized Gaussian space) is maximal. If X has a normal distribution the value X_d can be found from:

$$X_d = \mu_X - \alpha_X \beta_T \sigma_X \tag{2.2}$$

where μ_X and σ_X are the mean and standard deviation of X respectively, α_X is the (FORM) importance factor of the variable X with $0 \leq \alpha_X \leq 1$ for resistance properties and $-1 \leq \alpha_X \leq 0$ for load actions and β_T is the target reliability index. For other than normal distributions the value of X_d is found from $F(X_d) = \Phi(\alpha_X \beta_T)$ with $F(.)$ being the cumulative distribution function of the variable X.

In the Eurocodes ultimate limit states are generally verified by comparing the design value of the effects of actions E_d with the design value of the corresponding resistance R_d:

$$E_d \leq R_d \tag{2.3}$$

And hence, E_d and R_d should be determined in such a way that if eqn. (2.3) is met, the target reliability is fulfilled. In the current version of Eurocode 7, the design resistance is obtained by either applying partial factors to the characteristic values of the soil properties X_k, or to the resistances R or to both:

$$R_d = R\left\{\gamma_F F_{rep}; \frac{X_k}{\gamma_m};\ a_d\right\} \tag{2.4}$$
$$R_d = R\{\gamma_F F_{rep}; X_k;\ a_d\} / \gamma_R \tag{2.5}$$
$$R_d = R\left\{\gamma_F F_{rep}; \frac{X_k}{\gamma_m};\ a_d\right\} / \gamma_R \tag{2.6}$$

where the F_{rep} denote representative values of actions, X_k and γ_m (multiple) characteristic geotechnical strength parameters and corresponding partial factors , and the a_d design values of geometrical properties.The three equations represent different design approaches followed in the current EN 1997 for different structures and in different countries. The point here is only to show the role of characteristic values in the Eurocode design equations; for details on the design approaches and the remaining variables and symbols we refer to JRC (2021) or EN 1997 itself. In Dutch geotechnical practice, all three approaches are used for different structures.

The partial factors are ideally calibrated in such a way (in combination with the load factors) by e.g. selecting the proper α_X values, that the design or assessment decision for a large class of structures fulfils the reliability requirements with an acceptable level of accuracy. As the result (i.e. the target reliability is met given a structure fulfils the design values) must be applicable

for a large group of structures, the selected α_X values for code-calibration may be over-conservative in (a small minority of) individual cases.
It can be argued that achieving uniformity of the resulting reliability is not aided by the variety of different design approaches, which is one of the reasons that harmonization is one of the focal objectives in the development of the next-generation Eurocodes.

In this approach, the characteristic values are fixed quantiles of the actions and loads independent of consequence class and thus target reliability. Hence, the reliability differentiation for different target reliability levels is ensured by the partial factors. Relating partial factors to target reliabilities is not an objective in this report. Simple formulae, based on actually calculated or presumed influence coefficients in linearized stochastic reliability analyses, have been proposed already since the 1970's, e.g. (ISO,1973), (CIRIA, 1977) and (Toft Christensen and Baker, 1981), recently quoted in (Prästings et al., 2018). Yet, because influence coefficients inevitably vary broadly from one structure to another within the category for which a partial safety factor set is meant to be applicable, this approach needs further tuning, based on calibration analyses with representative test sets. This is e.g. shown in (Jongejan and Calle, 2014) and (Kanning et. al, 2016) in the course establishing codes for safety assessment of primary dikes in the Netherlands (WBI, 2017), and in (Stowa, 2009) for establishing the regional flood protection assessment code (LTI, 2017).

2.3 Definition of characteristic value

Regardless of the differences in design approach, the definition of the characteristic values in Dutch guidelines has followed the Eurocode definitions as stated in the following paragraphs of clause 2.4.5.2 from EN 1997-1:2004: (quote)

- (2) The characteristic value of a geotechnical parameter shall be selected as a cautious estimate of the value affecting the occurrence of the limit state.
- (11) If statistical methods are used, the characteristic value should be derived such that the calculated probability of a worse value […] is not greater than 5%.
- (7) The zone of ground governing the behaviour of a geotechnical structure at a limit state is usually much larger than a test sample or the zone of ground affected in an in-situ test. Consequently, the value of the governing parameter is often the mean of a range of values covering a large surface or volume of the ground. The characteristic value should be a cautious estimate of this mean value.
- (8) If the behaviour of the geotechnical structure at the limit state considered is governed by the lowest or highest value of the soil property, the characteristic value should be a cautious estimate of the lowest or highest value occurring in the zone governing the behaviour.

Notice that the Eurocodes generally adopt a Bayesian approach to uncertainties and degree-of-belief notion of probability (JRC, 2021). That is the reason why we do not refer to confidence intervals here, but to quantiles of a probability distribution which needs to contain all elements of uncertainty in the parameters of interest, including statistical uncertainty.

Paragraphs (7) and (8) are a more detailed specification of the notion of the 'value affecting the occurrence of the limit state' in (2). The general take-away is that if substantial spatial averaging is involved in a failure mode, the characteristic value refers to the mean value of the soil property in the affected volume. On the other hand, if no averaging is involved, the characteristic value refers to the lowest or highest 'point value'. With these definitions, EN 1997 makes a practical simplification by contemplating the upper and lower bounds for the 'value affecting the occurrence of the limit state'. The elaborations in this report will focus on the Eurocode definitions as stated and interpreted above, and will also describe how a more realistic amount of spatial averaging, rather than full or no averaging, can be quantified. Furthermore, we will focus on characteristic values for single material properties with no spatial

trends. Soil properties usually show a trend, especially in the depth direction. This should be dealt with detrending the data and only look at the statistics of the detrended data.
ISSMGE-TC304 (2021) contains a literature overview of approaches to define and operationalize characteristic values for a wider set of conditions and definitions, e.g.:

- soil properties with depth-trends
- multi-variate properties with correlations
- combination of site data with prior knowledge
- sophisticated approaches to account for the structural response

For some of the approaches it can be argued whether or not they meet the current Eurocode definitions. Yet, in our view, it is essential with probability-based approaches to characteristic values, to relate their degree of confidence to the overall design approach followed in each code of practice, including the target reliability and the partial factors. Characteristic values are just one of the ingredients in the overall approach and can, therefore, not be contemplated in isolation.

3 Modelling spatial variation of soil properties

3.1 Spatial variability of soil properties

In most current computation methods for geotechnical design or safety assessment, basic soil property values, such as shear strength parameters, volumetric weights, et cetera, are usually considered to be uniform within well distinguished soil units or layers of identical material type, such as sand, clay, Yet, outcomes of lab tests on soil samples, acquired from such units, or outcomes of in situ tests at different locations within the unit, show considerable variability. The use of simple univariate (independent random sample) statistics, to deal with it in a quantitative way in geotechnical design, have been suggested already from the mid 1960's, by (Lumb, 1966), (Schultze, 1971), (Wu & Kraft, 1970) and (Tang et al., 1976), to mention a few.

A widely held view was that variability of test results on spatially distributed acquired soil samples reflects mainly spatial variability of the soil property itself throughout the soil unit, probably due to fluctuations in space of deposition conditions. Only a limited part was viewed to originate from imperfections in the process of acquisition and handling test samples, and irreproducibility of test devices. (Cherubini, 1997) claims that up to roughly 30 percent of sample test result variances may be attributable to errors of this type. Although this is substantial, this still supports the idea that the main part of variability results from the deposition history.

Observed patterns of variability found using site characterization techniques such as cone resistance measurements (CPT's or SPT's) suggest a characteristic pattern of relatively rapid variation of soil strength in a vertical direction, relative to a constant mean or average trend with depth, besides much slower variations in a horizontal direction (Table A.2.1 in annex A.2). Spatial variability may have crucial effects on the behavior of soil volumes as a system, e.g. when considering resistance against slope failure. Yet, usually applied soil investigation does not reveal the pattern of spatial variability at a scale sufficiently detailed to evaluate these system effects in a deterministic way. Therefore, random field modeling was adopted, which enabled the ability to assess such effects. E.g. the effect of averaging of spatial fluctuation of soil strength over the surface or volume of a failure mode, needed in a definition of characteristic values, based on soil property testing, as elucidated in sections 4 and 5 of this report. And the effect of increase of failure along a dike or (or other type of embankment) as a function of the length of the dike or embankment section (in Dutch geotechnical and hydraulic engineering literature often referred to as length-effect)[1]. From the mid 1970's a stationary random field description was suggested in (Vanmarcke, 1977) and more elaborate in (Vanmarcke, 1983), which roughly captures the characteristics of the pattern type, shown in Figure 3.1; thus reflecting the impression of a series of CPT diagrams. Though a random field concept in geotechnical safety assessment was already suggested earlier, e.g. by Alonso (Alonso, 1976), Vanmarcke extended it with the elaboration of averaging (variance reduction) and length effects. As already explained in the introduction, this "basic" model needed extension.

Vanmarcke's basic random field model and the extended version will be described in section 3.2., including thoughts and decisions regarding choices of spatial correlation model type and parameters from available options. The extended version (composite model) is the basis for the analyses and elaborations in sections 4 and 5, thus presenting a stochastic/statistical basis for formulae to estimate characteristic values of geotechnical parameters, e.g. for dike slope

[1] *Essential for assessing target reliability indices of fdikes, in the Dutch Flood Defense codes, yet not further outlined in this report.*

safety assessment, from local or regional soil sampling and testing. An overview of these formulae, used in Dutch guidelines for design of primary and regional flood defenses, was provided in (Calle, 1996), however without the background presented here.

3.2 Random field model

3.2.1 Basic and composite model

The pattern of a soil property varying throughout a soil unit is modeled as a 3-D stationary random field. Roughly, this implies that the actual pattern of spatial variation of a soil property, e.g. the "drained cohesion" c, is considered to be a realization of random variables $c(x,y,z)$ in each location (x,y,z) of the soil unit, having identical probability distributions. For example, the normal distribution $N(\mu_c,\sigma_c)$, where μ_c is the expected mean and σ_c is the standard deviation. In this report x and z are taken to be spatial coordinates in a horizontal direction and y in the vertical direction, i.e. the width, length, and depth direction of a dike, as sketched in Figure 3.1. Furthermore, stationarity implies that autocorrelations among any two $c(x,y,z)$ and $c(x+\Delta x, y+\Delta y, z+\Delta z)$ is a function of the separation distance components only. Typical features of the spatially varying pattern of c may be achieved by choosing the autocorrelation function structure. For example:

$$\rho_c(\Delta x,\Delta y,\Delta z) = exp\left(-\left(\left(\frac{\Delta x}{d_{hx}}\right)^2 + \left(\frac{\Delta z}{d_{hz}}\right)^2 + \left(\frac{\Delta y^2}{d_v^2}\right)\right)\right) \tag{3.1}$$

where $\rho_c(\Delta x,\Delta y,\Delta z)$ equals the autocorrelation among the random variables $c(x,y,z)$ and $c(x+\Delta x, y+\Delta y, z+\Delta z)$. The parameters d_{hx} , d_{hz}. are correlation scale parameters, characterizing fluctuations of the soil property in the horizontal (x) and (z) directions, while d_v characterizes the fluctuation in the vertical (y) direction. Figure 3.1 shows the axes orientation. Also, it shows fluctuations in the horizontal direction are thought of as rather slow, reflecting spatial variation of deposit conditions, and fluctuations in the vertical direction are rapid, reflecting temporal variability of the deposition regime. Note that the σ_f in this figure must be interpreted as σ_c in the equations above, and $\tilde{c}(x_0,z)$ as μ_c , the notations in Figure 3.1 belong to the extended model, discussed below, which will further be referred to as the composite model.

Though the geological genesis of a soil deposit may justify the need of distinction between correlation decay in the two different horizontal, (x) and (z), directions, it is most often assumed that $d_{hx} = d_{hz} = d_h$, as will be assumed in the sequel in this report.

A key feature of the basic model is that "local averages" of the soil property, i.e. average values of the soil property at different locations (x,z) tend to be equal to μ_c, when thickness of the soil layer is large compared to the vertical correlation parameter d_v. The reason is, obviously, that differences in local averages are purely statistical of nature in this basic model and reduce as the depth of the soil layer increases. However, this turned out to be a weakness of the model. In the course of development of the Dutch code for river dike design (TAW, 1989), this random field model was adopted as a mathematical/statistical framework to derive statistically based characteristic soil properties. The idea was that such a framework would facilitate merging of various (local) sets of test data, obtained from sets of soil samples at different locations (along a dike), into one regional data model. This would enable derivation of characteristic geotechnical parameters for any location along the dike, either locations where local test data are available, or locations in between sampled locations. A basic requirement for merging of test data is, that geological conditions of locations where local test data were acquired, are reasonably similar, and well comparable with the conditions at "in between" locations, for which characteristic parameters derived from the regional dataset are meant to be applied.

It appeared, however, that variability among averages of the local test data sets at the different test locations, was significantly larger than could be explained from statistical uncertainties of local mean values in line with the basic random field model. The extension is illustrated in figure 3.1. In fact, it is defined as composite of the basic 3-D random field $f(x,y,z)$, superimposed on a 2-D (horizontal) field $\tilde{c}(x,z)$, reflecting slowly varying local additional means .Hence, the composite 3-D field is:

$$c(x,y,z) = \tilde{c}(x,z) + f(x,y,z) \tag{3.2}$$

where $\tilde{c}(x,z)$ is normally distributed, $N(\mu_c, \sigma_{\tilde{c}})$, and $f(x,y,z)$ is normally distributed, $N(0, \sigma_f)$, while it is assumed that the two fields are stochastically independent. Further, the autocorrelation functions are taken to be:

$$\rho_{\tilde{c}}(\Delta x, \Delta z) = exp\left(-\left(\frac{\Delta x}{d_h}\right)^2 - \left(\frac{\Delta z}{d_h}\right)^2\right) \tag{3.3}$$

And, similar to eqn. (3.1):

$$\rho_f(\Delta x, \Delta y, \Delta z) = exp\left(-\left(\left(\frac{\Delta x}{d_h}\right)^2 + \left(\frac{\Delta z}{d_h}\right)^2 + \left(\frac{\Delta y^2}{d_v^2}\right)\right)\right) \tag{3.4}$$

Note that identical horizontal correlation scale parameters have been assumed in eq. (3.3) and (3.4), although different choices of the d_h's in eqns. (3.3) and (3.4) would be possible. It is plausible that the assumption of identical scale parameters is a reasonable one, based on geological grounds.

Since the two components $f(x,y,z)$ and $\tilde{c}(x,z)$ are stochastically independent, the total variance of $c(x,y,z)$ equals:

$$\sigma_c^2 = \sigma_{\tilde{c}}^2 + \sigma_f^2. \tag{3.5}$$

where σ_c^2 may be estimated classically as sample variance ofthe regional dataset, while its components $\sigma_{\tilde{c}}^2$ and σ_f^2 must be determined elsewise Based on eqn. (3.5) we define the ratios:

$$\alpha = \frac{\sigma_f^2}{\sigma_c^2} \qquad \text{and consequently} \qquad (1-\alpha) = \frac{\sigma_{\tilde{c}}^2}{\sigma_c^2} \tag{3.6}$$

The ratio α is a crucial parameter of the composite random field model, the significant impact of it on determining characteristic values of a soil property will be shown in sections 4 and 5. Based on the eqns. (3.2) thru (3.6) the autocorrelation function of the composite random field model, as formally derived in Annex A.1, reads:

$$\rho_c(\Delta x, \Delta y, \Delta z) = exp\left(-\left(\frac{(\Delta z^2 + \Delta x^2)}{d_h^2}\right)\right)\left((1-\alpha) + \alpha\, exp\left(-\left(\frac{\Delta y^2}{d_v^2}\right)\right)\right) \tag{3.7}$$

Figure 3.1 shows an illustration of the composite model.

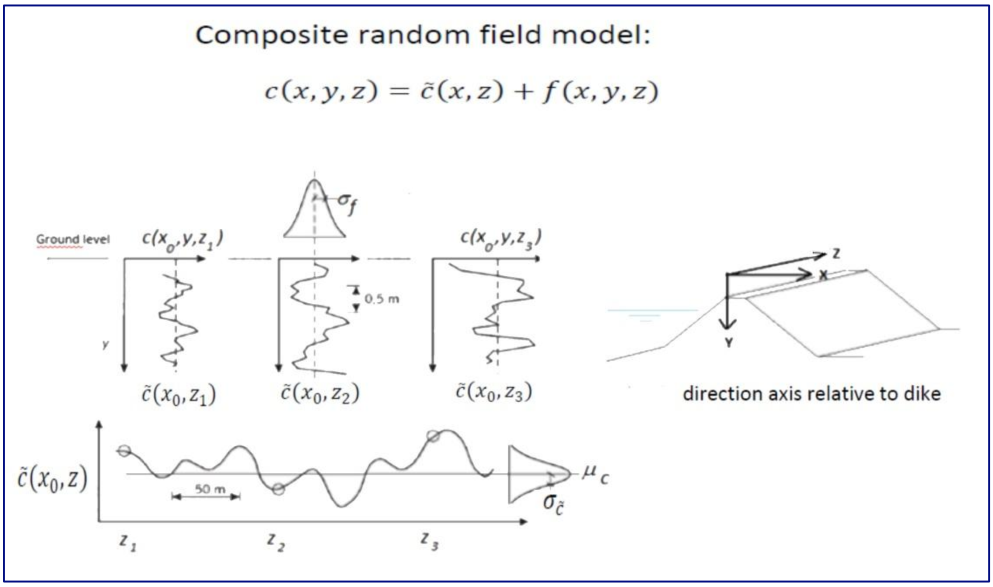

Figure 3.1. Characterization of the composite random field model..

Note that when α = 1, and consequently $\sigma_{\tilde{c}}$ = 0 (and thus $\sigma_c = \sigma_f$), the composite random field model reduces to the basic random field model, often referred to as Vanmarcke's model in literature, though the composite model was already suggested in (Vanmarcke, 1977). The elaboration given above was outlined in (Calle, 1990). The idea of creating regional test datasets by merging local test datasets, is explained in (Calle, 1996).

3.2.2 Choice of pdf and autocorrelation function type

Ideally, the choices of distribution type, its parameters, the autocorrelation function type and correlation parameters should be verified on the basis of data obtained from geotechnical site investigations. Most often, however, available data from usual soil investigations, is far from sufficient for significant statistical inferencing of model choices. Apart from basic parameters of the adopted probability distribution, decisions on model and parameter choices are based on "informed guessing", i.e. information from literature and intuitive deduction. Such decisions may therefore from a mathematical perspective not be unambiguous. From an engineering perspective, model choices should be made, which at the end serve the purpose of modeling the best, which, in our case is to derive plausible and sufficiently safe characteristic soil property values from (usually limited) measured data.

In case of positive valued random fields, the assumption of a log-normal distribution may be more logical, than the current assumption of a normal distribution, though the latter is necessarily less appropriate. Also, other distribution types have been suggested in geotechnical literature, e.g. based on high end statistics (skewness, kurtosis, etc.) of available data using theoretical decision diagrams, e.g. (Schultze, 1971). The inevitable truth is, however, that sample sizes of usual soil investigation in a project, are far too small for reliable use of these criteria. Hence the pdf-type is most often adopted, based on pragmatic issues and is frequently a choice between the normal or log-normal distribution only, discarding other options.

Lack of sufficient data is also (even more) an issue, when it comes to the choice of the autocorrelation function type and autocorrelation parameters. Different types of admissible autocorrelation function types have been suggested in literature, some of the most mentioned in literature are shown in Table 3.1; credit to the JCSS Probabilistic Model Code (Baker & Calle, 2006). Again, here the choice is not made on the basis of statistical inference, but by heuristic reasoning, which inevitably may include arbitrariness. For example, since soil property values are averages anyway over small volumes (lab test samples, or affected volumes of in situ test devices), one might expect that the spatial field of the soil property is a smoothly varying one. Its varying pattern is in fact a moving average over these small volumes. Hence the Gaussian autocorrelation function, type 3, is a convenient choice, since it fulfils a necessary condition for smoothness, namely that it is twice differentiable at zero lag ($\Delta x = 0$),(Papoulis, 1965). Other types in Table 3.1 would disqualify, on the basis of this reasoning, but might well not be rejected on the basis of statistical inference of soil property data from, even intensive, geotechnical investigation in practice.

Table 3.1: Admissible autocorrelation function types (non-exhaustive)

Type name	equation
1. Exponential	$\rho(\Delta x) = exp\left(-\frac{\lvert\Delta x\rvert}{d}\right)$
2. Exponential, oscillatory	$\rho(\Delta x) = exp\left(-\frac{\lvert\Delta x\rvert}{d}\right) cos(\omega\Delta x)$
3. Quadratic exponential (Gaussian)	$\rho(\Delta x) = exp\left(-\left(\frac{\Delta x}{d}\right)^2\right)$
4. Bilinear	$\rho(\Delta x) = \left(1-\frac{\lvert\Delta x\rvert}{d}\right) \quad for\ \lvert\Delta x\rvert \leq d$ $\rho(\Delta x) = 0 \quad for\ \lvert\Delta x\rvert > d$
Bilinear is applicable to 1-D fields only d and ω are correlation parameters d is frequently referred to as correlation length	

This also holds true for the correlation parameters. An impression of correlation parameter values, such as the correlation parameters d_v and d_h related scales of fluctuation δ_v and δ_h, is shown in Table A.2.1 in Annex A.2. The notion and use of scales of fluctuation is explained in this annex. The results in Table A.2.1 are indicative and presumably the outcomes of specific purpose soil investigation campaigns, involving large numbered sample sets of laboratory or in-situ tests. Usual soil investigation set ups in a geotechnical engineering project involve far from sufficient samples to determine correlation parameters. Thus, for these parameters we can only rely on literature-based indications.

As shown in Figure 3.1, adopted indications for the scale of fluctuation in the Dutch flood safety guidelines are 0.5 m, for the vertical, and 50 m for the horizontal scale of fluctuation. Note that these characterizations are expressed in terms of "scale of fluctuation", denoted as δ, while until now we used correlation parameter d. The notion "scale of fluctuation" has been introduced in (Vanmarcke, 1977) in order to enable better mutual comparison of the correlation parameters d, as in Table 3.1, for the various different correlation function types. This is further explained in detail in Annex A.2. For the type adopted in this report, type 3 in Table 3.1, the relation between the two is $\delta = d\sqrt{\pi}$. In Figure 3.1, the scales of fluctuation of $\delta_v = 0.5\,m$ and $\delta_h = 50\,m$ implicate autocorrelation parameters $d_v \approx 0.28\,m$ and $d_h \approx 28\,m$.

The concept of scale of fluctuation was of even more importance in comparing the "averaging effect" of a soil property in geotechnical limit state analyses. Vanmarcke demonstrated that the associated variance reduction, as function of the surface or volume dimensions of limit states, can be very well approximated by simple formulas, needing only scales of fluctuation, regardless the actually adopted autocorrelation function types from Table 3.1. See Annexes A.2 and A.3 for further details. Apparently, this makes a correct choice of autocorrelation function type less critical, as far as evaluation of the effect of averaging in the estimation of characteristic values is concerned (see sections 4 and 5 in this report).

Scales of fluctuation itself remain quite uncertain parameters. It will be shown in section 5, however, that simplifications must be adopted anyway, to derive practically tractable formulas for the determination of characteristic soil parameters from field data. The assumed scales of fluctuation, even though not very accurate, provide sufficient information to support the choice of simplifications.

3.2.3 Choice of the variance ratio parameter α; local vs regional test data

As explained, the composite random field model serves as a framework for statistical interpretation of a regional test dataset, created by merging local datasets, i.e. sets of test data from local soil investigations. The basic random field model, i.e. α = 1, was rejected for this merged data-model in the former Dutch river dike design code (TAW, 1989) since it did not well match with the available data from local test sets at hand then. The choice α = 0.75 was made, largely based on intuition; since there was no time, nor methodology and adequately structured data, available for rigorous (statistical) analysis. Of course, a classic variance ratio testing could have been performed, but this would yield smaller "prudent" estimates of α than held plausible anyway. The choice for α = 0.75 implies that 25% of the variance in a regional dataset is due to regional variations of the mean and 75% of the variance is due to variations around the local mean. Only some 15 years after its introduction, a statistical (semi) Bayesian approach was developed, which showed that this this intuitively adopted variance ratio is a plausible one (Calle, 2007) and (Calle, 2008), yet possibly somewhat on the conservative side. Though this approach could actually be helpful in deciding for each regional dataset which α –value assumption is suitable (and prudent from an engineering point of view), the initial choice in (TAW, 1989) remained unaltered in later guidelines for design and safety assessment, among which the most recent ones for design and safety assessment of primary dikes, (OI, 2014) (WBI, 2017), and regional dikes (LTV, 2015).

3.3 Non-uniformity of soil property: increasing trend with depth

Typically, soil strength properties involved in undrained slope stability assessment, increase (about linearly) in depth within in a soil layer. The previous "uniform" model can easily be extended with a linearly in depth varying expected mean value term, to capture this type of spatial variation. So, the expression (3.2) could become, for example:

$$c(x,y,z) = \tilde{c}(x,z;y_0) + b(y - y_0) + f(x,y,z) \qquad (3.8)$$

where b may, in the simplest extension of the random field model, be constant or of more complex nature, of course depending on the nature of variability of the field and the information available to determine it precisely. If we choose b (and y_0) to be a constant, we add two parameters to the model, but the stochastic character (pdf. and correlation structure of fluctuations, i.e. eqn. (3.7)) remains effectively unaltered. However, the parameter estimations from field data needs a somewhat more complex approach, because it involves linear regression calculus, as shown in (Calle, 1996). Also, the expression to determine the characteristic values of the soil property, eqn. (5.4), needs some extension, as illustrated in (Van Meekeren, 2019).

4 Variance reduction due to averaging

4.1 Spatial averaging

In geotechnical model evaluations in order to check the ultimate limit state (ULS) or the serviceability limit state (SLS), we typically use uniform soil properties in our geotechnical models despite the soil's spatially variable nature. Explicit random field modelling is still a method virtually limited to academic studies at the time of writing, and it certainly was in the last decades when the approaches for characteristic values described in this report were developed. From random finite element (RFEM) analyses we now know that the equivalent uniform geotechnical parameter statistics entail (a) spatial averaging of random fluctuations in the zone of influence of the failure mode in question, and (b) a reduced mean value due to failure surfaces being attracted to weak zones. The simplification that has been made hitherto is to consider spatially averaged statistics and to neglect the effect of attraction to weak zones, which arguably is only relevant or significant in specific cases (see 5.6).

Following the above, we consider the mean of a soil property over a certain volume or surface, reflecting the dimensions of a failure mode. However, soil property data available from soil testing reflects point data, i.e. related to small sample volumes acquired for lab testing or affected in in situ testing. For instance, for a slope stability computation we are interested in the mean and standard deviation of the shear strength of a layer, while tri-axial test point data are available. Since local weaker and stronger spots are all present within the volume, averaging occurs and the variance of the volume of lower than the variance of the point value. This is called averaging in this report. Hence, we are interested in determining the mean and variance of a volume, given we have point inputs. The amount of averaging depends on the considered volume and scale of fluctuation of the property.

In this report we consider a point property $c(x, y, z)$ within a soil layer. The amount of averaging is determined for a square prism, for which the volume is determined by width (B), length (L) and height (H). The spatial average (spatial mean) of $c(x, y, z)$ is given by eqn. (4.1):

$$\bar{c}_{B,H,L} = \frac{1}{B\,H\,L} \int_{x_0}^{x_0+B} \int_{y_0}^{y_0+H} \int_{z_0}^{z_0+L} c(x,y,z)\, dx\, dy\, dz \tag{4.1}$$

The expected mean value and variance of $\bar{c}_{B,H,L}$ can be derived as shown in Annex A.3. Its expected mean equals the expected mean of the field: $E\left(\bar{c}_{B,H,L}\right) = \mu_c$ and its variance equals:

$$\sigma^2_{\bar{c}_{B,H,L}} = E\left(\left(\bar{c}_{B,H,L} - \mu_c\right)^2\right) = \sigma_c^2\ \Gamma^2_{x,B}\, \Gamma^2_{z,L}((1-\alpha)\ +\ \alpha\, \Gamma^2_{y,H}) \tag{4.2}$$

In this equation, $\sigma^2_{\bar{c}_{B,H,L}}$ depends on the field variance, σ_c^2, on the variance ratio parameter α (since only the local fluctuations around the mean can average) and on the variance reduction factors $\Gamma^2_{x,B}$, $\Gamma^2_{z,L}$ and $\Gamma^2_{y,H}$ for respectively the width B in x-direction, length L in z-direction and depth H in y-direction of the volume involved in a (slope or other) failure, see for axis orientation Figure 3.1. Width, length and depth reflect dimensions of a (ULS- or SLS-) failure affected volume or surface, referred to as problem scale for short.

As an approximation, suggested by (Vanmarcke, 1977), in case of non-rectangular shaped volumes of failure modes, the dimensions B, L and H can roughly be taken to be the dimensions of the smallest axis-parallel rectangular body which includes this volume. The same holds when averaging along surfaces is concerned.

The variance reduction factors Γ^2 depend on the problem scales relative to the scales of fluctuation and will be discussed further in section 4.2.The term $\Gamma_{x,B}^2\,\Gamma_{z,L}^2$ in eqn. (4.2) reflects variance reduction due to the horizontal averaging effect and the term $\Gamma_{y,H}^2$ reflects variance reduction due to the vertical averaging effect. Note that in case of small failure mode dimensions, relative to the fluctuation scales, the Γ's equal 1, and thus $\sigma_{\bar{c}_{B,H,L}}^2 = \sigma_c^2$, so then there is no variance reduction at all. This is, e.g., of relevance in case the soil property is (random modeled) thickness of an impervious blanket layer on top of an aquifer, when evaluating safety from ground breach due to heave.

4.2 Approximation of variance reduction factors (Vanmarcke 1977)

The exact computation of variance reduction factors requires solving multi integrals, examples of which are shown in eqns. (A.3.9) to (A.3.11) in the Annex A.3. Based on the elaboration of these integrals for the various autocorrelation function-types, shown in Table 3.1. (Vanmarcke, 1977) derived a simple approximation for the variance reduction factor.

$$\Gamma_{x,B}^2 \approx 1 \;\text{ when }\; B \le \delta_h \qquad (4.3)$$
$$\Gamma_{x,B}^2 \approx \frac{\delta_h}{B} \;\text{ when }\; B > \delta_h$$

Note the use of, δ_h, the scale of fluctuation. In this report $\delta_h = d_h\sqrt{\pi}$, where d_h is the correlation parameter in eqn. (3.1). Similar relations are, of course, valid for $\Gamma_{z,L}^2$ and $\Gamma_{y,H}^2$. The crux of Vanmarcke's approximation is that the relation (4.3) is valid for all of the autocorrelation-decay-function-types shown in Table (3.1), provided adequate determination of the scales of fluctuation from the correlation parameters, as shown in Table A.2.2 in Annex A.2.

5 Characteristic values in Dutch codes and guidelines

5.1 Characteristic values

As mentioned in section 2.3, a statistically determined characteristic value of a geotechnical parameter, in a computational ULS or SLS evaluation, must be a cautious estimate of the parameter value affecting the occurrence of the limit state, with (calculated) probability of 5% that a worse value is plausible. The geotechnical parameter in the computation model often represents an average of the spatially varying soil property, over an affected volume of surface involved in the occurrence of the limit state failure. Hence, in terms of averaged soil properties over a failure mechanism affected volume $B \times H \times L$ in the previous section, the characteristic geotechnical parameter (further referred to as characteristic value in this section) would read (see Annex A.4):

$$\begin{aligned} \bar{c}_{B,H,L,kar} &= \mu_c \pm 1.645 \sqrt{\sigma^2_{\bar{c}_{B,H,L}}} \\ &= \mu_c \pm 1.645\, \sigma_c \sqrt{\Gamma^2_{x,B}\, \Gamma^2_{z,L}\left((1-\alpha) + \alpha\, \Gamma^2_{y,H}\right)} \end{aligned} \tag{5.1}$$

under the provision that the (stationary) random field variable c is Gaussian. The expected mean value μ_c and standard deviation σ_c must usually be derived from a sample test[2] on small volumes, either in lab or in situ. So, for these parameters only estimations are available $\hat{\mu}_c$ and s_c, derived from the sample test outcomes and statistical uncertainty of these estimates must be taken into account. The detailed elaboration in Annex A.4 yields the following expression:

$$\bar{c}_{B,H,L,kar} = \hat{\mu}_c \pm t^{0.95}_{N-1}\, s_c \sqrt{\left(\Gamma^2_{x,B}\, \Gamma^2_{z,L}\left((1-\alpha) + \alpha\, \Gamma^2_{y,H}\right) + \frac{1}{N}\right.} \tag{5.2}$$

where:

- $\hat{\mu}_c$ is the estimation of μ_c, which equals the mean value of the test sample outcomes, $\bar{c}$ (cf. eqn. (A.4.2) in Annex A.4) when samples can be assumed uncorrelated (and unbiased). When samples can in essence not be assumed uncorrelated, eqn. (A.5.3) in Annex 5 should be used to establish the estimator.
- s_c is the estimation of σ_c , determined cf. eqn. (A.4.7) for uncorrelated/weakly correlated samples, and cf. eqn. (A.5.4) for essentially correlated samples.
- $t^{0.95}_{N-1}$ is the 95 percentile value of the Student t probability distribution with $N-1$ degrees of freedom, to account for statistical uncertainty of s_c
- Variance reduction factors $\Gamma^2_{x,B}$, $\Gamma^2_{z,L}$, and $\Gamma^2_{y,H}$, see Section 4.
- Ratio of local and regional variance α, see Section 3.
- N sample size of the (local or regional) dataset.

Statistical uncertainty of $\hat{\mu}_c$ (estimator for μ_c) is accounted for by the term $\frac{1}{N}$ while statistical uncertainty of s_c (estimator for σ_c) is accounted for by replacing the standard Gaussian 95 percentile $\xi^{0.95}$ = 1.645 by the Student $t^{0.95}_{N-1}$. As for the sample size, i.e. number of independent

[2] Sample test is assumed to be free of spatial trends and tests are assume free of measurement error. Spatial trends need a different approach, not further elaborated here. Measurement errors will be dealt with in section 5.3.

test-outcomes, note that a weak assumption of independency of sample outcomes is briefly discussed in annex A.5.

5.2 Simplifications adopted in Dutch Guidelines and Regulations

The variance reductions factors (Γ^2) depend on estimated dimensions of potential ULS or SLS failure affected volumes or surfaces on the one side, and estimated scales of fluctuation on the other. Both are affected with uncertainty and/or inaccuracy. Dimensions of potential failure affected soil volumes or surfaces can only roughly be estimated on beforehand. As for scales of fluctuation, even very extended and systematic soil investigation setups will yield only rough estimations. So at this point we have to rely for a great part on empirics and literature. Thus, soft approximate data, which, nevertheless, is useful for practice, if we use it to derive cautious, but sensible, prescriptions for characteristic values of soil parameters.

5.2.1 Simplification for averaging over large volumes

The amount of variance reductions, expressed in terms of the variance reduction factors, highly depends on this size of the considered volume in the various directions (horizontal, vertical) in relation to the scales of fluctuation in these respective directions. As mentioned in section 3.2, an indication of the scales of fluctuation for the horizontal and vertical is $\delta_h = 50\,m$ and $\delta_v \approx 0.5\,m$ in accordance with Figure 3.1., in case the soil property $c(x,y,z)$ reflects shear strength, to be used in, e.g. slope stability computations.
In case of a slope failure of an embankment or dike, typical longitudinal dimensions (z-direction in see Figure 3.1) in practice may range from $L \approx 30\,m$ to $L \approx 70\,m$, while widths (in x-direction) may be typically $B \approx 10 - 20\,m$. Given a horizontal scale of fluctuation of $\delta_h = 50\,m$, this implies the variance reduction factors $\Gamma^2_{x,B}$ and $\Gamma^2_{z,L}$ likely to be not much smaller than 1.0. Eqn. (4.3) yields:

$$\Gamma^2_{z,L} \approx 1 - 0.7 \qquad \text{and} \qquad \Gamma^2_{x,B} \approx 1 \tag{5.3a}$$

The vertical dimension (H) of a slope failure is typically in the order of several meters for embankments, which is relatively large compared to the vertical scale of fluctuation $\delta_v \approx 0.5\,m$. Using this, and $H = 3 - 4\,m$, then eqn. (4.3) yields:

$$\Gamma^2_{y,H} \approx 0.16 - 0.12 \tag{5.3b}$$

As argued previously, estimated scales of fluctuation are mainly indicative especially for horizontal scales of fluctuation. At the best they could be determined with some accuracy from extensive and specially structured field sampling, which, however, will likely be lacking in every day geotechnical projects. Hence, values of scales of fluctuation, used in practice, must be adopted from reported special research, as e.g. referenced in the Annex A.2. The assumption $\delta_h \approx 50\,m$ in Figure 3.1 is reasonably comparable with the range, suggested in Table A.2.1 for soil shear strengths. However, the assumption $\delta_v \approx 0.5\,m$ is relatively small, compared to indications in this table. This assumption is based on the observed typical variation pattern in recorded CPT-diagrams in soft clay and peat layers in the Netherlands. Based on the ranges, suggested in eqns. (5.3a) and (5.3b), the following choice was adopted in the Dutch flood safety related design and assessment guidelines:

$$\Gamma^2_{x,B} = 1 \qquad \Gamma^2_{z,L} = 1 \qquad \text{and} \qquad \Gamma^2_{y,H} = 0 \tag{5.3c}$$

in which it is speculated that the effect of ignoring horizontal variance reduction, i.e. $\Gamma^2_{z,L} = 1$, is compensated by the effect of exaggerating the vertical variance reduction, i.e. $\Gamma^2_{y,H} = 0$. This reduces eqn. (5.2) to the expression:

$$\bar{c}_{B,H,L,kar} \approx \hat{\mu}_c - t_{N-1}^{0.95}\, s_c \sqrt{(1-\alpha) + \frac{1}{N}} \tag{5.4}$$

Note that "±" in eqn. (5.2) is replaced by "-" in eqn. (5.4), since shear strength is concerned and hence a low characteristic value (5 percentile). Eqn. (5.4) with (α = 0.75), i.e. the composite random field model, using regionally merged data over a large area, was first introduced in the Dutch guideline for river dike design (TAW, 1989). Later on this was re-adopted in the previously mentioned Dutch guidelines for safety assessment of primary and regional flood defenses.

When use is made of only acquired data at a relatively small location, then α = 1, i.e. the basis random field model (see 4.2), is most likely a reasonable assumption. The idea is that the variance ratio α will gradually reduce from 1 to 0.75 and may be even less, as the area, over which acquired local data(sets) are merged, increases from a relatively small locality to an extent area. This looks a reasonable assumption, but further research is needed to implement such approach.

As for now, we adopt the basic random field model α = 1, in case of data acquisition at a relatively small locality, and the composite model with α = 0.75 for merged local datasets, regardless the extent of the area over which test data is merged into a regional dataset. The α = 0.75 is considered to be a prudent approach in the Dutch guidelines, although there is, strictly speaking, no evidence to exclude smaller values of this model parameter.
For large sample sizes, i.e. $N \longrightarrow \infty$, eqn. (5.4) reduces to:

$$\bar{c}_{B,H,L,kar} \approx \hat{\mu}_c - 1.65\, s_c \sqrt{(1-\alpha)} \tag{5.5}$$

thus for a *local* dataset, i.e. α = 1 (and $N \longrightarrow \infty$) eqn. (5.4) reduces to:

$$\bar{c}_{B,H,L,kar} \approx \hat{\mu}_c \tag{5.6}$$

5.2.2 Simplification for averaging over small volumes

For point mechanisms with relatively limited volumes of affected soil by the ULS, there will also be limited variance reduction in vertical direction. This is, for example, the case in pile tip resistance. In these cases, $\Gamma_{y,H}^2$ = 1 (besides $\Gamma_{x,B}^2$ = 1 and $\Gamma_{z,L}^2$ = 1) is likely a realistic assumption, which reduces eqn. (5.2) to:

$$\bar{c}_{B,H,L,kar} \approx \hat{\mu}_c \pm t_{N-1}^{0.95}\, s_c \sqrt{1 + \frac{1}{N}} \tag{5.7}$$

This is valid for both local ($\alpha = 1$) and regional ($\alpha < 1$) datasets.

5.3 Measurement uncertainty

Up to now it was assumed that the dataset variance s_c^2 reflects only spatial variability of the soil property. Most likely, however, observed lab or in situ test results may contain errors of various sources. As previously mentioned, according to (Cherubini, 1997) up to 30 percent of the variance may be due to measurement errors, irreproducibility of test devices, disturbed soil samples, etc. The question is if and how this uncertainty should be taken into account.
The assumption made here about measurement errors is that they are mutually independent for all of the measurements. Systematic deviation should, when known, be accounted for by correcting measurement outcomes. When not known, correction is of course not an option. A Bayesian type of uncertainty approach might be helpful then, but this will not further be explored here. Correction in case of known systematic deviation would rather affect the estimated mean value of the soil parameter instead of its variance, which we focus on here. Therefore, we will further investigate the assumption of independent measurement errors.

These could be well modelled as a random variable with a normal distribution with expected mean value equal to zero and variance equal to a certain percentage, say Cherubini's "30 pct." of the (local or regional) test data variance. Then the terms under the square root symbol in eqn. (5.2) would apply to 70 % of the test data variance, since only this part is attributable to spatial variability. However, there will be an additional term, similar to the term for statistical uncertainty, to cover the 30 % measurement uncertainty. Applying this, eqn. (5.4) becomes now:

$$\bar{c}_{B,H,L,kar} \approx \hat{\mu}_c - t_{N-1}^{0,95}\, s_c \sqrt{0.7(1-\alpha) \; + \; 0.7\frac{1}{N} + 0.3\frac{1}{N}}$$

$$= \hat{\mu}_c - t_{N-1}^{0,95}\, s_c \sqrt{0.7(1-\alpha) \; + \; \frac{1}{N}} \qquad (5.8)$$

Hence, this yields a higher estimated characteristic strength value compared to eqn. (5.4), which is also true for any other fraction between zero and the assumed 0.3 here. This is due to the averaging of all the independent measurement errors. So, unless it is quite certain which part of the test data variance is at least attributable to (random) errors, induced by the process of sample acquisition, preparation and testing, it is advisable from a prudent engineering point of view not to account for these errors in the estimation of characteristic values of soil strength properties. In case of a local data set (α = 1), the effect of independent measurement errors disappears, as both the measurement error and the spatial variability components average.

5.4 Statistical inference from spatially correlated data

When inferring statistical geotechnical parameters, based on data, often the (implicit) assumption is made that soil samples are uncorrelated. However, if they are close enough together, a partial correlation might be present according to the underlying spatial model, which might have to be corrected for in the inference. This is further elaborated in Annex A.5, where it is also argued that in practice there will not be much difference between computed regional means and variances with or without explicitly accounting for assumed random field associated spatial correlations. However, it is better to avoid having many correlated data, in order to avoid the need to account for correlation as discussed in Annex A.5, which can be achieved by a careful design of the sampling plan based on the presumed spatial model of the soil, or by pre-processing of the data.

5.5 Lognormal distribution

As pointed out in section 3.2.2 an alternative to the assumption of normality of a soil property distribution, is the assumption of lognormality, which is an effective way to avoid negative, and thus unrealistic, estimations of characteristic soil strength parameters. Lognormality of some stochastic soil property field $c(x,y,z)$ implies that the logarithms $\log\big(c(x,y,z)\big)$ have a normal distribution by definition. Formula's for characteristic values of "point" values (small volumes) may easily be obtained, based on this definition, see eqn. (8) in (Calle, 1996). Yet, deriving formula's for determination of characteristic values of averages over larger volumes, similar to the adopted line of thought in sections 4 and 5 is not trivial. Contrary to normally distributed fields, where an average value over some volume is again normally distributed, the average over some volume of a log normal field is not log normally distributed. However, it may well be assumed that the simplification at the end for normally distributed fields, eqn. (5.3.c), is valid, or at the least reasonably assumable, for log normal fields too. This implies that the characteristic value of a (local) expected mean may simply be obtained from statistics of the logarithms of observed strength values.

The often used equation for a characteristic value for a lognormal distribution (e.g. Calle, 1996) has the form of eqn. (5.9) for full spatial averaging of local variability. It should be noted that this is the characteristic value of the **median** of a lognormal distribution.

$$c_{med,kar} = e^{\left\{(\ln c)_{av} - \frac{t_{N-1}^{0.95} s_{\ln(c)}}{\sqrt{N}}\right\}} \tag{5.9}$$

where $(\ln c)_{av}$ is the mean value of the set of natural logarithms of observed values of a soil property c and $s_{\ln(c)}$ is the standard deviation of this set. . The characteristic value of the expected **mean** of the log normal distribution is, approximately:

$$\bar{c}_{kar} \approx e^{\left\{(\ln c)_{av} + \frac{s_{\ln(c)}^2}{2} - \frac{t_{N-1}^{0.95} s_{\ln(c)}}{\sqrt{N}}\right\}} \tag{5.10}$$

Whether eqn. (5.9) or eqn. (5.10) should be used depends on the actual system behavior. In case of doubt, eqn. (5.9) should be used since this is the more conservative one.

5.6 Concluding remarks

The parameter α of the composite random field model significantly affects estimates of characteristic soil strength parameters. Estimation of this parameter for a particular regional dataset is a tedious task, while significant uncertainty remains. However, it looks that the engineering intuition based $\alpha \approx 0.75$ in (TAW, 1979) was a reasonable choice (Calle, 2007/2008).

Yet in most published proposals relating to computation of characteristic soil parameter values, the basic Vanmarcke-model, i.e. α = 1, was adopted (Schneider & Schneider, 2013, Ching et al.,2020, Hicks et al., 2019, to mention a few). It can be seen from eqns. (5.4), (5.5) and (5.8), that this is not a prudent, and possibly even unsafe, assumption, when applied in connection with a regional data set, for actually not sampled locations (locations in between sampled areas within a region). A reassessment on the basis of the composite random variation model with α = 0.75 would surely be of interest, to compare these proposals with the current Dutch flood safety guidelines.

The assumed scales of fluctuation in this report are indicative. Yet they seem to agree well with other published material and are, in the end, only used (and in fact usable) to support simplifications, to achieve simple formula's for characteristic value calculations, based on sample data alone.

A fairly new approach to estimate characteristic soil parameters is proposed in (Hicks et al., 2019) using an RFEM (Random Finite Element Method). This approach looks promising, in the sense that it inherently accounts for spatial variability induced effects, though more developments and testing will be needed for use in every day practice.

6 Conclusion

Characteristic values of soil properties are determined either based on engineering judgement or on a statistical basis. As stated in the introduction, the authors recognized persisting confusion about some concepts and assumptions underlying the approach to characteristic values as implemented in Dutch guidelines and codes of practice. To remedy this situation, and to support future developments in approaches to characteristic values, this report documents the theoretical (probability-based) backgrounds and the practical assumptions made in the implementation. Below we provide some final reflections.

The definition of characteristic values is closely related to the required reliability targets in the Eurocodes and in the safety standards for flood defenses through the semi-probabilistic verification format with partial factors. This implies that the operational definitions of characteristic values always need to be considered in conjunction with the overall safety or reliability concept for limit state verification in a guideline or code of practice.

We have highlighted the importance of considering spatial variability in soil properties to the degree that that is applicable for the structure and limit state in question. In this report, this was achieved by applying spatial averaging. The equations currently stated in Dutch guidelines consider upper and lower bound approaches in this regard, i.e. either point properties (no averaging) or layer averages (full averaging) is considered. We have shown which assumptions have been made to arrive at these simplifications, based on a more sophisticated and generic elaboration of spatial averaging.

Merely looking at the equations for characteristic values, these may seem rather simplistic. Hopefully, the discussions in this report have demonstrated that the fundamental basis of the equations is rather wide-ranging in considerations, such as:

- spatial variability and averaging
- regional versus local
- statistical uncertainty
- measurement error
- correlated versus uncorrelated data

It is noteworthy that recently numerous proposals have been made to operationally define characteristic values, also for conditions deviating from the elaborations in this article (i.e. stationary random fields). For example, soil parameters with depth trends or correlated parameters require different treatment. An overview of these developments can be found in ISSMGE-TC304 (2021).
We believe that for any improvement or alternative proposals to operationally define characteristic values, it is crucial to address all these issues. Of course, that does not mean that all aspects need be ultimately part of the recipe. As we have shown, also the current formulations work with simplifications based on (mostly judgement-based) assumptions. Yet is seems important to us that all essential elements are covered in the justification.

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A ANNEX: Mathematical background and proofs

A.1 Proof of autocorrelation function eq. (3.7) in main text

Assuming normality of the probability distributions[3] of $f(x,y,z)$ and $\tilde{c}(x,z)$ in section 3, the random field property value $c(x,y,z)$ can be written as

$$c(x,y,z) = \mu_c + t(x,z)\sigma_{\tilde{c}} + r(x,y,z)\sigma_f \tag{A.1.1}$$

for any spatial point (x,y,z) within the soil layer. Herein μ_c is the so called regional expected mean value (or regional mean), $\sigma_{\tilde{c}}$ the standard deviation of local means $\tilde{c}(x,z)$ at location (x,z)) from the regional mean μ_c. Further, $t(x,z)$ and $r(x,y,z)$ are standard normal mutually independent random field functions, defined as:

$$t(x,z) = \frac{\tilde{c}(x,z)-\mu_c}{\sigma_{\tilde{c}}} \tag{A.1.2}$$

and

$$r(x,y,z) = \frac{f(x,y,z)}{\sigma_f} \tag{A.1.3}$$

Both with expected mean value equal to 0 and standard deviation equal to 1, and auto-correlation functions according to the assumptions in section 3:

$$\rho_t(\Delta x, \Delta z) = \rho_{\tilde{c}}(\Delta x, \Delta z) = exp(-\frac{\Delta x^2+\Delta z^2}{d_h^2}) \tag{A.1.3}$$

and

$$\rho_r(\Delta x, \Delta y, \Delta z) = \rho_f(\Delta x, \Delta y, \Delta z) = exp\left(-\frac{\Delta x^2+\Delta z^2}{d_h^2} - \frac{\Delta y^2}{d_v^2}\right) \tag{A.1.4}$$

Where d_h en d_v denote the parameters of autocorrelation decay in horizontal and vertical direction respectively.

The expected mean value $E[c(x,y,z)]$ of $c(x,y,z)$ according the notation in eq. (A.1.1) is, obviously, equal to μ_c (because expected means $E[t(x,z)]$ and $E[r(x,y,z)]$ both equal zero). Notably $E[.]$ denotes the expectation operator, which is linear functional operator. The variance $E[(c(x,y,z) - E[c(x,y,z)])^2]$ is, also expectedly:

$$\begin{aligned} &E[(c(x,y,z) - E[c(x,y,z)])^2] = E[(c(x,y,z) - \mu_c)^2] = \\ &\quad E\left[t^2(x,z)\sigma_{\tilde{c}}^2 + 2\,t(x,z)\sigma_{\tilde{c}}\; r(x,y,z)\,\sigma_f + r^2(x,y,z)\sigma_f^2\right] = \\ &\quad E\left[t^2(x,z)\right]\sigma_{\tilde{c}}^2 + 2\,E[t(x,z)r(x,y,z)]\;\sigma_{\tilde{c}}\,\sigma_f + E[r^2(x,y,z)]\,\sigma_f^2\, \right] = \\ &\qquad \sigma_{\tilde{c}}^2 + \sigma_f^2 = \sigma_c^2 \end{aligned} \tag{A.1.5}$$

because $E[t(x,z)r(x,y,z)] = 0$ (the random functions $t(x,z)$ and $r(x,y,z)$ are mutually independent) and because $E[t^2(x,z)]$ and $E[r^2(x,y,z)]$ both equal 1.
The autocorrelation function of $c(x,y,z)$, which is a only a function of distance components $\Delta x, \Delta y, \Delta z$ between any two spatial points in the soil layer equals:

[3] *Throughout the Annex, as in the main text, it is assumed that random field variables c, f and $\bar{c}$ have normal (Gaussian) probability distributions*

$$\rho_c(\Delta x,\Delta y,\Delta z) \stackrel{\text{def}}{=} \frac{E[\,c(x,y,z)\;c(x+\Delta x,y+\Delta y\;,z+\Delta z\;)]}{\sigma_c^2} = \frac{[\,E[t(x,z)\;t(x+\Delta x,z+\Delta z\;)]\;\sigma_{\tilde{c}}^2\;+E[r(x,y,z)\;r(x+\Delta x,y+\Delta y,z+\Delta z)]\;\sigma_f^2\;]}{\sigma_c^2}$$

and because expectations of cross terms $E[t(.)r(.)]$ equal zero, we find, based on the assumptions (A.1.3) and (A.1.4) that:

$$\rho_c(\Delta x,\Delta y,\Delta z) = (1-\alpha)\,exp\left(-\frac{\Delta x^2+\Delta z^2}{d_h^2}\right) + \alpha\,exp\left(-\frac{\Delta x^2+\Delta z^2}{d_h^2}-\frac{\Delta y^2}{d_v^2}\right) = exp\left(-\frac{\Delta x^2+\Delta z^2}{d_h^2}\right)\left((1-\alpha)+\alpha\,exp\left(-\frac{\Delta y^2}{d_v^2}\right)\right) \qquad (A.1.6)$$

This concludes the proof of eqn. (3.7) in the main text.

A.2 Scales of fluctuation (cf. Vanmarcke, 1977)

Several autocorrelation-function types and actually suggested scales of fluctuation have been proposed in literature throughout the years, since the notion of random field modeling of soil properties was introduced in the mid 1970's. Table A.2.1, adopted from (Baker & Calle, 2006), provides an illustrating, though not exhaustive, overview of literature over the past decades. The recently published state-of-the-art review ISSMGE-TC304 (2021) contains additional information and sources.

Table A.2.1: Scales of Fluctuation from various sources

Source	***Soil property***	***Purpose***	***Spatial model type***	***Scale of Fluctuation***
Tang 1979	Marine clay, average cone resistance (CPT) from 0-3m below sea bottom. Different levels	Design skirts offshore platform	Gaussian	*δ_h = 55 m* *δ_h = 35 – 60 m*
Asoaka et al 1982	Undrained shear strength	Modeling vertical spatial variability	Exponential	*δ_v = 2.5 – 6 m*
Mulla 1988	Surface temperature Water content Penetrometer resistance Sand content (sandy clay) Clay content	Prediction of water content	Semi variogram spherical	*δ_h = 50 – 70 m* *40 – 60 m* *40 – 70 m* *60 – 80 m* *40 – 60 m*
Ronold 1990	Shearing strength (clay)	Capacity of tension piles	Gaussian	*δ_v = 2 m*
Unlu et al 1990	ln($K_{unsarurated}$) Soil parameter (unspec) Water capacity	Comparison study		*$\delta_{h,lnK}$ = 12–16 m* *$\delta_{h,par}$ = 40 m* *$\delta_{h,cap}$ = 12–16 m*
Soulié et al 1990	Shear strength	Modeling spatial variability dam design	Exponential	*δ_v = 2 m* *δ_h = 20 m*
Rehfeldt et al 1992	log Permeability	Modeling spatial variability, tracer tests	Exponential	*Flowmeter:* *δ_v = 3.2 m,* *δ_h = 25 m* *Several tests:* *δ_v = 1.5-3 m* *δ_h = 25–50 m*
Honjo et al 1991	Unconfined compr. strength	Slope stability evaluation	Exponential	*δ_v = 4 m* *δ_h = 80 m*

Rosenbaum 1987	Thickness of natural deposit		Variogram, spherical	*δ_h = 750 m*
Hess et al 1992	ln Permeability	Contaminant migration	Exponential	*δ_v = 0.2–1 m* *δ_h = 2–10 m*
Chiasson 1995	CPT, vane shear strength	Modeling spatial variability	Variogram, spherical	*δ_v = 1.5 m*
Vrouwenvelder & Calle 2003	CPT, avg. cone resistance deep glacial sands	Modeling spatial variability	Gaussian	*δ_h = 20–35 m*

Note that the spatial parameters in this table refer to "scales of fluctuation". This parameter definition has been introduced in (Varmarcke, 1977), to enable a comparison of correlation length parameters used in combination with different autocorrelation function types may have a different meaning. Vanmarcke's definition of scale of fluctuation is:

$$\delta = 2 \int_0^{\infty} \rho(\tau) \; d\tau \qquad \text{(A.2.1)}$$

Application to the autocorrelation function types of Table 3.1 in the main text yields the relations between scale of fluctuation δ and correlation parameter d, given in Table A.2.2

Table A.2.2: Relation between scale of fluctuation and correlation

Type	Scale of fluctuation δ
1. Exponential	$\delta = 2d$
2. Exponential, oscillatory	$\delta = \frac{2d}{1+\omega^2 d^2}$
3. Gaussian	$\delta = d\sqrt{\pi}$
4. Bilinear	$\delta = d$

A.3 Variance reduction due to averaging: Proof of eqn. (4.2) of main text

Soil properties in a geotechnical ULS or SLS verification analysis usually represent averages of the soil property over a surface or in a volume involved in the limit state definition. Depending on the sizes of such surfaces or volumes, variances of average values of the soil property over these surfaces or volumes will be less than variances of "point" values itself. This is called variance reduction. We will illustrate this using the following example. Suppose $c(x,y,z)$ is a soil property, modeled as a random field, the average value of which over a rectangle volume $\{x,y,z\}$, with $(x_0 \le x \le x_0 + B)$, $(y_0 \le y \le y_0 + H)$ and $(z_0 \le z \le z_0 + L)$ is a parameter in some ULS or SLS verification analysis. The average value of $c(x,y,z)$ is:

$$\bar{c}_{B,H,L} = \frac{1}{B\,H\,L} \int_{x_0}^{x_0+B} \int_{y_0}^{y_0+H} \int_{z_0}^{z_0+L} c(x,y,z)\, dx\, dy\, dz \qquad \text{(A.3.1)}$$

Following the expression adopted in eqn. (A.1.1), it can easily be seen that the expected mean value $E[\bar{c}_{B,H,L}]$ equals μ_c and the variance $\sigma^2_{\bar{c}_{B,H,L}}$ equals:

$$\sigma^2_{\bar{c}_{B,H,L}} = E\left[\left(\bar{c}_{B,H,L} - \mu_c\right)^2\right] =$$

$$E\left[\left(\frac{1}{B\,H\,L} \int_{x_0}^{x_0+B} \int_{y_0}^{y_0+H} \int_{z_0}^{z_0+L} t(x,z)\sigma_{\bar{c}} + r(x,y,z)\sigma_f \; dx\, dy\, dz\right)^2\right] = * \qquad \text{(A.3.2)}$$

The utmost right-hand side term of eqn. (A.3.2) will be elaborated further. For this we use the following properties of integrals:

$$\left(\int f(x)dx\right)^2 = \left(\int f(x)dx\right)\left(\int f(\xi)d\xi\right) = \iint f(x)f(\xi)dx\, d\xi \qquad \text{(A.3.3)}$$

and, since it is a linear operator:

$$E[\int f(x)dx] = \int E[f(x)]dx \tag{A.3.4}$$

Further we use an abbreviate notation for the integrant in eqn. (A.3.2):

$$t(x,z)\sigma_{\bar{c}} + r(x,y,z)\sigma_f \Rightarrow by\ notation\ \ G(x,y,z) \tag{A.3.5}$$

Herewith, the elaboration of eqn. (A.3.2) becomes:

$$* = E\left[\frac{1}{(B\ H\ L)^2}\ \left(\int_{x_0}^{x_0+B}\int_{y_0}^{y_0+H}\int_{z_0}^{z_0+L} G(x,y,z)\,dx\,dy\,dz\right)^2\right] =$$

$$\frac{1}{(B\ H\ L)^2}\int_{x=x_0}^{x_0+B}\int_{\xi=x_0}^{x_0+B}\int_{y=y_0}^{y_0+H}\int_{\eta=y_0}^{y_0+H}\int_{\tau=z_0}^{z_0+L}\int_{z_0}^{z_0+L} E[\,G(x,y,z)\,G(\xi,\eta,\tau)]dxd\xi\,dyd\eta\,dzd\tau \tag{A.3.6}$$

Further elaboration of the integrant yields:

$$\begin{aligned} &E[\,G(x,y,z)\,G(\xi,\eta,\tau)] = \\ &\quad E\left[(t(x,z)\sigma_{\bar{c}} + r(x,y,z)\sigma_f)(t(\xi,\tau)\sigma_{\bar{c}} + r(\xi,\eta,\tau)\sigma_f)\right] = \\ &\quad \sigma_{\bar{c}}^2\,E[t(x,z)\,t(\xi,\tau)] + \sigma_f^2\,E[r(x,y,z)\,r(\xi,\eta,\tau)] = ** \end{aligned} \tag{A.3.7a}$$

and, using the expressions (A.1.3) and (A.1.4))

$$\begin{aligned} &** = \sigma_{\bar{c}}^2\,\rho_t(x-\xi,\ z-\tau) + \sigma_f^2\,\rho_r(x-\xi, y-\eta, z-\tau) = \\ &\sigma_{\bar{c}}^2\ exp\left(-\left(\frac{x-\xi}{d_h}\right)^2\right)exp\ \left(-\left(\frac{z-\tau}{d_h}\right)^2\right) \\ &\quad + \sigma_f^2\ exp\left(-\left(\frac{x-\xi}{d_h}\right)^2\right)ex\,p\left(-\left(\frac{y-\eta}{d_v}\right)^2\right)\ exp\ \left(-\left(\frac{z-\tau}{d_h}\right)^2\right) \end{aligned} \tag{A.3.7b}$$

Substitution of (A.3.7b) in (A.3.2) and (A.3.6) yields, finally:

$$\begin{aligned} \sigma_{\bar{c}_{B,H,L}}^2 = &\left(\frac{1}{B^2}\int_{x=x_0}^{x_0+B}\int_{\xi=x_0}^{x_0+B} exp\left(-\left(\frac{x-\xi}{d_h}\right)^2\right)\,dxd\xi\right) \\ \text{x} &\left(\frac{1}{L^2}\int_{\tau=z_0}^{z_0+L}\int_{z_0}^{z_0+L} exp\left(-\left(\frac{z-\tau}{d_h}\right)^2\right)\,dzd\tau\right) \\ \text{x} &\left(\sigma_{\bar{c}}^2 + \sigma_f^2\ \left(\frac{1}{H^2}\int_{y=y_0}^{y_0+H}\int_{\eta=y_0}^{y_0+H} exp\left(-\left(\frac{y-\eta}{d_v}\right)^2\right)\,dyd\eta\right)\right) \end{aligned} \tag{A.3.8}$$

The integrals in eqn. (A.3.8) are written for short as:

$$\left(\frac{1}{B^2}\int_{x=x_0}^{x_0+B}\int_{\xi=x_0}^{x_0+B} exp\left(-\left(\frac{x-\xi}{d_h}\right)^2\right)\,dxd\xi\right) = \Gamma_{x,B}^2 \tag{A.3.9}$$

$$\left(\frac{1}{L^2}\int_{\tau=z_0}^{z_0+L}\int_{z_0}^{z_0+L} exp\left(-\left(\frac{z-\tau}{d_h}\right)^2\right)\,dzd\tau\right) = \Gamma_{z,L}^2 \tag{A.3.10}$$

$$\left(\frac{1}{H^2}\int_{y=y_0}^{y_0+H}\int_{\eta=y_0}^{y_0+H} exp\left(-\left(\frac{y-\eta}{d_v}\right)^2\right)\,dyd\eta\right) = \Gamma_{y,H}^2 \tag{A.3.11}$$

and are referred to as variance reduction factors. It can easily be seen that these factors are positive and less than or equal to 1. Using these abbreviations and the relations:

$$\sigma_{\bar{c}}^2 = (1-\alpha)\,\sigma_c^2 \text{ and } \sigma_f^2 = \alpha\,\sigma_c^2 \qquad \text{(A.3.12)}$$

we find the expression for the variance of the mean value of the soil property c within the volume $BxLxH$.

$$\begin{aligned}\sigma_{\bar{c}_{B,H,L}}^2 &= \sigma_c^2\left((1-\alpha)\,\Gamma_{x,B}^2\,\Gamma_{z,L}^2 + \alpha\,\Gamma_{x,B}^2\,\Gamma_{z,L}^2\,\Gamma_{y,H}^2\right)\\ &= \sigma_c^2\,\Gamma_{x,B}^2\,\Gamma_{z,L}^2\left((1-\alpha) + \alpha\,\Gamma_{y,H}^2\right)\end{aligned} \qquad \text{(A.3.13)}$$

This concludes the proof of eqn. (4.2) in the main text.

A.4 Characteristic values of soil properties, Proof of eqn. (5.1) of main text

In Dutch geotechnical guidelines, characteristic values of soil properties are to represent 5%-lower or 95%-upper bounds of the soil property values to be used in ULS- or SLS-based computation models. In most of these models, these soil property values represent average values of the soil property over the ULS- or SLS- affected volume or surface. Uncertainties involved in the estimation of characteristic values are aleatory and statistical of nature. The variance in eqn. (A.3.13) represents the aleatory part of uncertainty. If the basic random distribution parameters, i.e. expected mean value μ_c and variance σ_c^2 are known, or statistical estimation uncertainty is discarded, characteristic values (in case of a normal distribution) can be determined from the formula:

$$\begin{aligned}\bar{c}_{B,H,L,kar} &= \mu_c \pm 1.645\sqrt{\sigma_{\bar{c}_{B,H,L}}^2}\\ &= \mu_c \pm 1.645\,\sigma_c\sqrt{\Gamma_{x,B}^2\,\Gamma_{z,L}^2((1-\alpha) + \alpha\,\Gamma_{y,H}^2)}\end{aligned} \qquad \text{(A.4.1)}$$

For simplicity of notation, the expression under the square root symbol in this equation will temporarily be denoted as G, so:

$$\bar{c}_{B,H,L,kar} = \mu_c \pm 1.645\,\sigma_c\sqrt{G} \qquad \text{(A.4.1a)}$$

Usually the expected mean value, μ_c, must be estimated from the available local or regional lab or in situ test datasets. When it can be assumed that the test outcomes $\{c_i\}$ $(i = 1 \dots N)$ of such a dataset are stochastically independent[4] and free of bias, then the average value of the set:

$$\bar{c} = \frac{1}{N}\sum_{i=1}^{N} c_i = \hat{\mu}_c \qquad \text{(A.4.2)}$$

is an unbiased estimator, $\hat{\mu}_c$, for μ_c, meaning that in expectation the set average $\bar{c}$ equals μ_c. Its variance is (in case c follows a Gaussian distribution):

[4] *Within a stationary (non- white noise) random field, correlation among field points is specified by the spatial autocorrelation structure. In the estimation of the expected mean value, eqn. (A.4.2), and the variance, eqn. (A.4.7), mutual autocorrelation among $\{c_i\}$ should therefore be taken into account in these equations. Further explanation follows in paragraph A.4.3.*

$$\sigma_{\bar{c}}^2 = \frac{1}{N}\,\sigma_c^2 \qquad \text{(A.4.3)}$$

Likewise eqn. (A.1.1) we find the expression for the unknown μ_c:

$$\mu_c = \hat{\mu}_c + \xi\,\sigma_{\bar{c}} = \hat{\mu}_c + \xi\,\sigma_c\sqrt{\frac{1}{N}} \qquad \text{(A.4.4a)}$$

where ξ is a standard normally distributed random variable, $N(0{,}1)$, and for the “ULS/SLS-parameter” $\bar{c}_{B,H,L}$:

$$\bar{c}_{B,H,L} = \mu_c + \eta\,\sigma_c\sqrt{G} \qquad \text{(A.4.4b)}$$

where η is a standard normal random variable. Combination of eqns. (A.4.4a) and (A.4.4b) yields:

$$\bar{c}_{B,H,L} = \mu_c + \eta\,\sigma_c\sqrt{G} = \hat{\mu}_c + \xi\,\sigma_c\sqrt{\frac{1}{N}} + \eta\,\sigma_c\sqrt{G} \qquad \text{(A.4.4c)}$$

In this equation ξ and η are standard normally distributed and mutually independent, random variables. Similar to the elaboration resulting in eqn. (A.1.5), the eqn. (A.4.4c) yields the standard deviation of $\bar{c}_{B,H,L}$, including the effect of statistical uncertainty involved in the estimation of μ_c from the dataset. So:

$$\sigma_{\bar{c}_{B,H,L}} = \sigma_c\sqrt{\frac{1}{N} + G} \qquad \text{(A.4.5)}$$

From which the expression for (respectively, high and low) characteristic value of $\bar{c}_{B,H,L}$ follows:

$$\bar{c}_{B,H,L,kar} = \hat{\mu}_c \pm 1.645\,\sigma_c\sqrt{G + \frac{1}{N}} \qquad \text{(A.4.6)}$$

where 1.645 is the 95 percentile of the standard normal probability function.

Like the estimate for the expected mean value, also the (local or regional) standard deviation σ_c (or variance σ_c^2) must be determined from the available (local or regional) test sample dataset. The test sample variance:

$$s_c^2 = \frac{1}{(N-1)}\sum_{i=1}^{N}(c_i - \hat{\mu}_c)^2 \qquad \text{(A.4.7)}$$

(whether the test set is local or regional) is an unbiased estimator, meaning that in expectation $E[s_c^2] = \sigma_c^2$, however it suffers from statistical uncertainty, which is accounted for correctly by replacing in eqn. (A.4.6) the σ_c by s_c and the standard normal distribution associated 95-percentile deviation, 1.645, by the Student t distribution associated 95 percentile deviation, $t_{N-1}^{0,95}$. So, the formula for the characteristic value, (A.4.6), changes into:

$$\bar{c}_{B,H,L,kar} = \bar{c} \pm t_{N-1}^{0,95}\,s_c\sqrt{G + \frac{1}{N}} \qquad \text{(A.4.8)}$$

Replacing *G* again by its original expression, this yields:

$$\bar{c}_{B,H,L,kar} = \hat{\mu}_c \pm t_{N-1}^{0,95}\, s_c \sqrt{(\, \Gamma_{x,B}^2\, \Gamma_{z,L}^2 ((1-\alpha) \;+\; \alpha\, \Gamma_{y,H}^2) + \tfrac{1}{N}} \qquad \text{(A.4.9)}$$

This concludes the proof of eqn. (5.2) of the main text.

A.5 Statistics, based on spatially correlated data

Test data, as the $\{c_i\}$ $(i = 1 \ldots N)$ in the previous paragraph, reflect observations from test samples (lab or in situ) acquired from spatial locations within the random field considered. As we adopted a spatial structure of this field, i.e. the autocorrelation-function, the test data to determine the parameters of the field should be considered mutually correlated accordingly. Suppose the test data are associated with the locations where data was acquired, $\{c_i(x_i, y_i, z_i)\}$ $(i = 1 \ldots N)$, than the following mutual correlations between any two of the test data should be taken into account:

$$\rho(c_i\,, c_j) = \rho_c(x_i - x_j, y_i - y_j, z_i - z_j) = \rho_c(\Delta x, \Delta y, \Delta z) \qquad \text{(A.5.1)}$$

where $\rho_c(\Delta x, \Delta y, \Delta z)$ is the adopted autocorrelation function. For convenience we denote these correlations as $\rho_{i,j}$; they are represented in matrix notation as:

$$\boldsymbol{R} = \begin{pmatrix} \rho_{1,1} & \cdots & \rho_{1,N} \\ \vdots & \ddots & \vdots \\ \rho_{N,1} & \cdots & \rho_{N,N} \end{pmatrix} \qquad \text{(A.5.2)}$$

where $\boldsymbol{R}$, called correlation matrix of the test dataset, is a square matrix of rank N, symmetrical, with diagonal terms equal to 1 and off diagonal terms < 1 . The eqn. (A.4.2) to estimate the expected mean value of the field then reads:

$$\hat{\mu}_c = \left(\underline{e}^T \boldsymbol{R}^{-1} \underline{e}\right)^{-1} \left(\underline{e}^T \boldsymbol{R}^{-1} \underline{c}\right) \qquad \text{(A.5.3)}$$

where $\hat{\mu}_c$ is again an unbiased estimator of the (local or regional) expected mean value μ_c , $\boldsymbol{R}^{-1}$ de inverse of the correlation matrix, $\underline{e}$ a vector of size N with elements equal to 1 and $\underline{c}$ a vector of size N with elements equal to c_i $(i = 1 \ldots N)$. Further is $\underline{e}^T$ the transposed form of vector $\underline{e}$. The estimator for the variance σ_c^2 becomes:

$$s_c^2 = \frac{1}{(N-1)} \left(\underline{c} - \underline{e}\,\hat{\mu}_c\right)^T \boldsymbol{R}^{-1} \left(\underline{c} - \underline{e}\,\hat{\mu}_c\right) \qquad \text{(A.5.4)}$$

It can be shown that $\hat{\mu}_c$ is an unbiased estimator for the expected mean value μ_c and that s_c^2 is an unbiased estimator for the variance σ_c^2 , moreover, as the number of acquired samples N increases, the variances of $\hat{\mu}_c$ and s_c^2 decrease, asymptotically to zero. Proof is held in (Calle, 2007). It can easily be seen that in case there is no correlation among the test samples, thus $\boldsymbol{R} = \boldsymbol{I}$, the unity matrix, eqns. (A.5.3) and (A.5.4) reduce to eqns. (A.4.2) and (A.4.7). Moreover, it may be expected that differences between the outcomes of eqns. (A.4.2) and (A.5.4) and between eqns. (A.4.7) and (A.5.4) are not substantial, in case of a usual set up and intensity of geotechnical site investigation. Thus, the use of eqns. (A.4.2) and (A.4.7), disregarding any effect spatial correlation, stemming from the adopted random field model, may have only limited effect on correct estimation of expected mean value and variance of the field. Yet, it should be realized that this effect may become important in case of densely sampled field measurements.

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Bijlage 3: GEOLIB Tutorial 1: Deriving Lognormal statistics from measurements

https://deltares.github.io/GEOLib-Plus/latest/community/probabilistic_macrostability/Tutorial_1_Fit_Lognormal_Distribution.html

Tutorial 1: Deriving Lognormal statistics from measurements

This tutorial shows how to derive parameters of the Lognormal distribution using the Method of Moments, and by doing statistics on the logarithm of the values $ln(x)$. The tutorial covers the calculation of characteristic values, inclusing statistical uncertainty and (spatial) averaging. The following steps are discussed: 1. Importing the dataset, interpreting the data 2. Determine the sample mean and standard deviation 3. Determine the Lognormal distribution parameter using the Method of Moments 4. Determine the characteristic values 5. Determine the inputs for D-Stability, taking into account the statistical uncertainty

Step 1: Importing the dataset, and interpreting the data.

In the next cells, we import the python packages necessary to run this tutorial. If the packages are not available, install them in your Python environment using `pip install <package_name>`

```
import numpy as np
import scipy.stats as stats
import pandas as pd
import seaborn as sns
import matplotlib.pyplot as plt
```

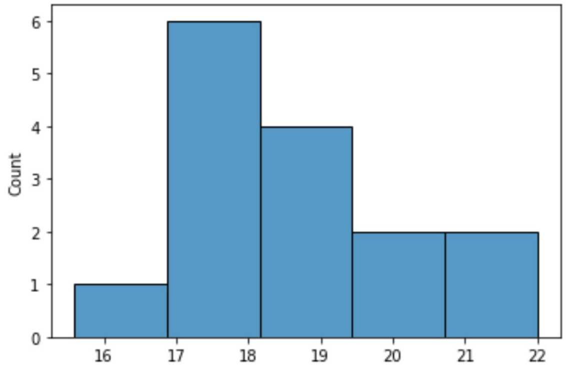
Import the data using the Pandas package, and show the first 5 entries of the dataframe with the measurements

```
# The headers are automatically taken as column names.
df = pd.read_csv("input_volumetricweight_testresults.txt", sep=",")
df
```

	VolWeight
0	17.17
1	18.16
2	19.05
3	17.50
4	17.57
5	20.26
6	18.64
7	18.35
8	17.18
9	15.58
10	17.35
11	19.52
12	22.01
13	17.36
14	21.16

Get our measurements in a numpy array `x`, and plot the histogram of the data.

```
x = df['VolWeight'].to_numpy()
sns.histplot(x, bins=5)
plt.show()
```



Step 2: Sample mean and sample standard deviation of the data set

Step 2a: Calculate the mean and standard deviation of the 'real' values

```
# First set up a function to calculate.
def calculate_mean_and_std(x):
    sample_mean = np.mean(x)
    sample_std = np.std(x, ddof=1)
    cov = sample_std/sample_mean

    print('Sample mean = \t\t\t%0.2f'%(sample_mean))
    print('Sample std = \t\t\t%0.3f'%(sample_std))
    print('Coefficient of variation cov = \t%0.3f'%(cov))

    return sample_mean, sample_std

# Now calculate the mean and std with that function
print('Mean and standard deviation of x')
mu, sigma = calculate_mean_and_std(x)
```

```
Mean and standard deviation of x
Sample mean =                     18.46
Sample std =                      1.701
Coefficient of variation cov =    0.092
```

Step 2b: Calculate the mean and standard deviation of the logarithmic values

```
# Here we do almost exactly the same, however, we
# calculate the mean and std of log(x)
print('Mean and standard deviation of ln(x)')
m, s = calculate_mean_and_std( np.log(x) )
```

```
Mean and standard deviation of ln(x)
Sample mean =                     2.91
Sample std =                      0.090
Coefficient of variation cov =    0.031
```

Step 3: Fitted Lognormal Distribution using method of

moments (MoM)

First we introduce two functions.

1. The first `lognormal_parameters_from_mu_sigma(mu_x, sigma_x, shift)` transforms the mean and standard deviation of x (i.e. the first and second moment, respectively) into the parameters of the Lognormal distribution.
2. The second `mu_sigma_from_lognormal_parameters(mu_y, sigma_y, shift)` does the exact opposite: it transforms the parameters of the Lognormal distribution into the mean and standard deviation of x.

```
def lognormal_parameters_from_mu_sigma(mu, sigma, shift=0):
    """
    Convention: a Lognormal distribution of X,
    means that Y = ln(X) is normally distributed.
    INPUT
        mu = mean(X)
        sigma = std(X)
        shift = default 0
    OUTPUT
        m = mean(Y)
        s = stdev(Y)
        shift
    """
    m = np.log( (mu - shift)**2 / np.sqrt(sigma**2 + (mu - shift)**2) )
    s = np.sqrt( np.log(sigma**2 / (mu - shift)**2 + 1))
    return m, s, shift

def mu_sigma_from_lognormal_parameters(m,s,shift=0):
    """
    Convention: a Lognormal distribution of X,
    means that Y = ln(X) is normally distributed.
    INPUT
        m = mean(Y)
        s = std(Y)
        shift = default 0
    OUTPUT
        mu = mean(X)
        sigma = std(X)
        shift = default 0
    """
    mu = np.exp(m + s**2 / 2) + shift
    sigma = np.sqrt((mu - shift)**2 * (np.exp(s**2) - 1))
    return mu, sigma, shift
```

To transform the mean and standard deviation (the first and second moment, respectively) into the parameters of the Lognormal distribution, we call the function. Then we print the result.

```
# Assume the lognormal is not shifted (shift=0)
shift=0
m, s, shift = lognormal_parameters_from_mu_sigma(mu, sigma, shift)
print('Location, scale, and shift parameters of the\
Lognormal distribution (Y=log(X)) are: \n\
m_Y = \t\t%0.2f\n\
s_Y = \t\t%0.3f\n\
shift_Y =\t%0.1f '%(m,s,shift))
```

```
Location, scale, and shift parameters of theLognormal distribution (Y=log(X)) are:
m_Y =               2.91
s_Y =               0.092
shift_Y =   0.0
```

The next step is to check the other formula for 'the other way around' (parameters of the Lognormal distribution -> mean and standard deviation).

```
# Assume the lognormal is not shifted (shift=0)
shift=0
mu, sigma, shift = mu_sigma_from_lognormal_parameters(m,s,shift)
print('Mean, standard deviation, and shift parameters of the\
Lognormal distribution are:\n\
mu_X = \t\t%0.2f\n\
sigma_X = \t%0.3f\n\
shift_X =\t%0.1f \n'%(mu,sigma,shift))
```

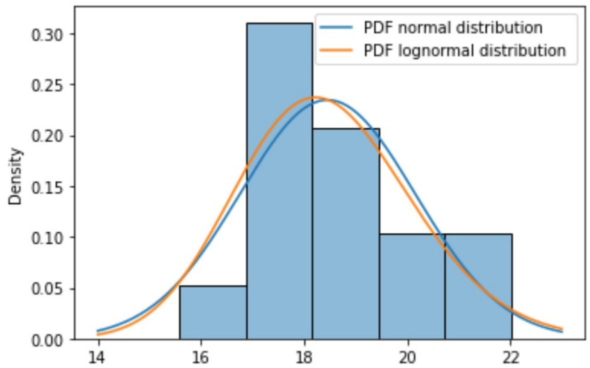
```
Mean, standard deviation, and shift parameters of theLognormal distribution are:
mu_X =              18.46
sigma_X =   1.701
shift_X =   0.0
```

Step 3c: Visual inspection of the results

Now we can visulize the results. We first set up two functions for drawing the pdf of a normal and a lognormal. Then we call this function to make the plots we want.

```
# First set up
def plot_pdf_normal(ax, x_range, mu, sigma):
    '''
    Calculate the pdf values using the norm.pdf function.
    Note that python uses the following terminology:
      location (loc) = mean value
      scale (scale) = standard deviation
    '''
    pdf_values = stats.norm.pdf(x_range, loc=mu, scale=sigma)
    ax.plot(x_range, pdf_values, label='PDF normal distribution')
    ax.set_label('Probability density (pdf)')

def plot_pdf_lognormal(ax, x_range, m, s, shift=0, label=''):
    '''
    Calculate the pdf values using the lognorm.pdf function.
    Note that python uses the following terminology:
      loc (location) = shift
      scale (scale) = median value (where cdf=0.5 ) -> exp(m)
      s (standard deviation) = s
    '''
    pdf_values = stats.lognorm.pdf(x_range, s=s, loc=shift, scale=np.exp(m))
    ax.plot(x_range, pdf_values, label='PDF lognormal distribution '+label)
    ax.set_label('Probability density (pdf)')
    ax.set_ylim(0)
    ax.legend(loc=1)


# Plotting the histogram of the data
ax = sns.histplot(df, bins=5, stat='density')

# Set up an x range for plotting
x_range = np.linspace(14,23,200)
```

```
# Plot the pdf of the normal and lognormal distribution for
# the xrange, using the above function.
plot_pdf_normal(ax, x_range, mu, sigma)
plot_pdf_lognormal(ax, x_range, m, s)
```

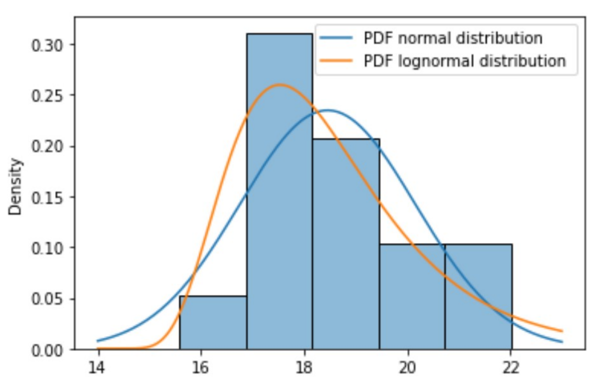


Optional step 3d: Derive parameters for a shifted Lognormal distribution, with all values shifted to 14

Note: this is an advanced step to show how a (manually shifted) lognormal distribution could be derived. To this end, we first shift all our data points 'to the left' with a value of 14 (assuming 14 is some kind of physical lower limit for this parameter). Then we derive the statistics for the lognormal parameter, and parse the shift=14 parameter to the plots.

```
# We assume 14 is a physical lower limit for our volumetric weight parameter
shift = 14
# Shift our measurements and derive the lognormal statistics.
m_, s_ = calculate_mean_and_std( np.log(x - shift) )

# Plot the histogram and the pdf of a shifted Lognormal distribution
ax = sns.histplot(df, bins=5, stat='density')
plot_pdf_normal(ax, x_range, mu, sigma)
plot_pdf_lognormal(ax, x_range, m_, s_, shift)
```

```
Sample mean =                    1.42
Sample std =                     0.401
Coefficient of variation cov =   0.282
```



Step 4: Characteristic values

Step 4a: Calculate the characteristic value with the Student-T factor.

The number of measurements influences the statistical uncertainty of our estimated mean and standard deviation. Therefore we apply the Student-T factor to account for statistical uncertainty due to a small number of samples. The characteristic value is calculated as follows:

$$X_{char,5\%} = \exp(m + t_{5\%,n-1} \cdot s \cdot \sqrt{1 + 1/n})$$

with m and s the lognormal parameters μ_Y and σ_Y.

```
n = len(df)
student_t_factor = stats.t.ppf(0.05, df=n-1) # df = degrees of freedom, which is n-1
print('t-value \t\t%0.2f'%(student_t_factor))

# The 5%-characteristic value, using the t distributions, including
# the statistical uncertainty in the estimated mean (s/sqrt(n))
# Note the + sign before the student_t_factor, because student_t_factor is negative
characteristic_value = np.exp(m + student_t_factor * s * np.sqrt( 1 + 1 / n ) )
print("5p-char_value = \t%0.2f"%(characteristic_value))
```

```
t-value             -1.76
5p-char_value =     15.55
```

Step 4b: Calculate the characteristic value with and without spatial avaraging

To account for spatial averaging the Γ^2 (variance reduction) factor is introduced into the equation

$X_{char.,5\%} = \exp(m + t_{5\%,n-1} \cdot s \cdot \sqrt{\Gamma^2 + 1/n})$ with m and s the lognormal parameters μ_Y and σ_Y.

Default assumptions on this factor are: - $\Gamma^2 = 1.0$: point values, no spatial averaging - $\Gamma^2 = 0.25$: estimate of the regional variability, assumption half of the variance averages. - $\Gamma^2 = 0.0$: spatial average, full spatial avaraging

```
# Allocate a list to store multiple outcomes
stored_values = []

# Calculate the characteristic value for different choises of spatial averaging:
gamma2_values = [1.0, 0.25, 0.0];
for gamma2 in gamma2_values:
    characteristic_value = np.exp(m + student_t_factor * s * np.sqrt( gamma2 + 1 / n ) )
    print("gamma2: 5p-char_value = \t%0.2f"%(characteristic_value))
    stored_values.append(characteristic_value)
```

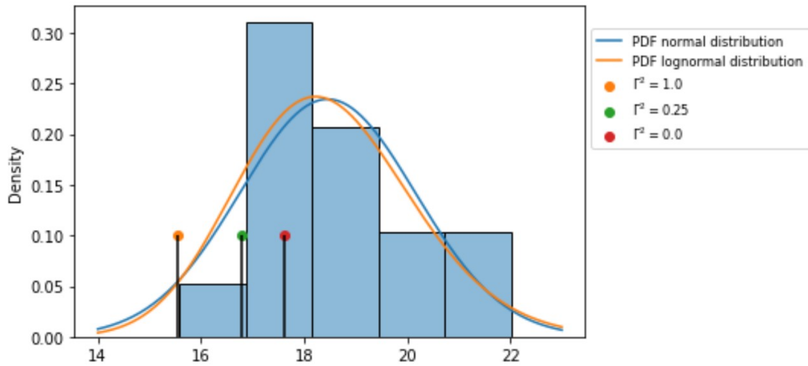
```
gamma2: 5p-char_value =     15.55
gamma2: 5p-char_value =     16.78
gamma2: 5p-char_value =     17.63
```

Step 4c: Visually checking the results

The results are visually checked with the histogram and the pdf's, showing the calculated characteristic value for the different asssumptions with a black vertical line and a colored dot.

```
ax = sns.histplot(df, bins=5, stat='density')
plot_pdf_normal(ax, x_range, mu, sigma)
plot_pdf_lognormal(ax, x_range, m, s)

for gamma2, value in zip( gamma2_values, stored_values):
```

```
    plt.vlines(value, ymin=0, ymax= 0.1, color='k')
    plt.scatter(value, 0.10, label="$\Gamma ^{2}=%s$"%(gamma2) )

plt.legend(fontsize='small', bbox_to_anchor=(1,0.95))
plt.show()
```



Step 5: Get the inputs for D-Stability, taking into account the statistical uncertainty

To correct the Lornormal standard deviation for the statistical uncertainty, we increase the std with a factor.

This factor equals $t_{5\%,n-1}/z_{5\%}$, where $z_{5\%} = -1.645$.

Step 5a: For the uncertainty in the point values

```
correction_factor = student_t_factor / -1.645
s_point_incl_stat =  s * correction_factor * np.sqrt(1 + 1 / n)

mu, sigma, shift = mu_sigma_from_lognormal_parameters(m,
                                                      s_point_incl_stat,
                                                      shift=0)
print('Mean, standard deviation, and shift parameters of the\
Lognormal distribution \n\
For D-Stability input:\n\
mu_X = \t\t%0.2f\n\
sigma_X = \t%0.3f\n\
shift_X =\t%0.1f \n'%(mu,sigma,shift))
```

```
Mean, standard deviation, and shift parameters of theLognormal distribution
For D-Stability input:
mu_X =              18.47
sigma_X =   1.883
shift_X =   0.0
```

Step 5b: For the uncertainty in the regional average

```
s_regional_average_incl_stat =  s * correction_factor * np.sqrt(0.25 + 1 / n)

mu, sigma, shift = mu_sigma_from_lognormal_parameters(m,
                                                      s_regional_average_incl_stat,
                                                      shift=0)
print('Mean, standard deviation, and shift parameters of the\
Lognormal distribution \n\
```

```
For D-Stability input:\n\
mu_X = \t\t%0.2f\n\
sigma_X = \t%0.3f\n\
shift_X =\t%0.1f \n'%(mu,sigma,shift))
```

```
Mean, standard deviation, and shift parameters of theLognormal distribution
For D-Stability input:
mu_X =              18.41
sigma_X =   1.021
shift_X =   0.0
```

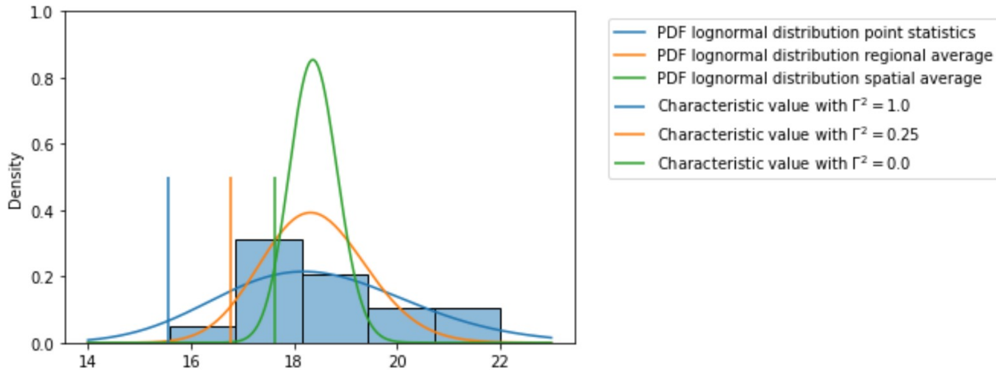
Step 5c: For the uncertainty in the spatial average

```
s_spatial_average_incl_stat =  s * correction_factor * np.sqrt(1 / n)

mu, sigma, shift = mu_sigma_from_lognormal_parameters(m,
                                                      s_spatial_average_incl_stat,
                                                      shift=0)
print('Mean, standard deviation, and shift parameters of the\
Lognormal distribution \n\
For D-Stability input:\n\
mu_X = \t\t%0.2f\n\
sigma_X = \t%0.3f\n\
shift_X =\t%0.1f \n'%(mu,sigma,shift))
```

```
Mean, standard deviation, and shift parameters of theLognormal distribution
For D-Stability input:
mu_X =              18.39
sigma_X =   0.467
shift_X =   0.0
```

Step 5d: Visualize the results

```
ax = sns.histplot(df, bins=5, stat='density')
plot_pdf_lognormal(ax, x_range, m, s_point_incl_stat,
                   label='point statistics')
plot_pdf_lognormal(ax, x_range, m, s_regional_average_incl_stat,
                   label='regional average')
plot_pdf_lognormal(ax, x_range, m, s_spatial_average_incl_stat,
                   label='spatial average')

colors=['tab:blue', 'tab:orange', 'tab:green']
for i, (gamma2, value) in enumerate(zip( gamma2_values, stored_values)):
    plt.vlines(value, ymin=0, ymax=0.5, color=colors[i], label="Characteristic value with $\
plt.legend(bbox_to_anchor=(1.05,1))
plt.ylim([0,1])
plt.show()
```


1.0
0.8
0.6
0.4
0.2
0.0
Density
14
16
18
20
22
PDF lognormal distribution point statistics
PDF lognormal distribution regional average
PDF lognormal distribution spatial average
Characteristic value with Γ² = 1.0
Characteristic value with Γ² = 0.25
Characteristic value with Γ² = 0.0

Bijlage 4: (Log)normale kansverdeling

Bijlage 5: Spreadsheet lineaire regressie c′ en φ′ uit eentraps triaxiaalproeven (Deltares)

Bijlage 6: Spreadsheet lineaire regressie c′ en φ′ uit eentraps DSS-proeven (Deltares)

Bijlage 7: Spreadsheet lineaire regressie S en m uit sterkteproeven

Bijlage 8: Spreadsheet lineaire regressie S en POP uit sterkteproeven

Bijlage 9: Spreadsheet lineaire regressie su-tabel en m uit sterkteproeven